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Investigating the effect of weight restrictions on estimating returns to scale in convex technologies

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Abstract

Abstract: Data envelopment analysis is a non-parametric method based on mathematics, which is used to evaluate the performance of a set of decision-making units in a production technology with multiple inputs and outputs. The idea of returns to scale is also one of the important topics in Data envelopment analysis is considered as the ratio of changes in output to inputs and is divided into three types: fixed, increasing and decreasing. The use of added weight restrictions in different models is a common method to reduce input flexibility and output weight. is, in the model with weight restrictions, weight restrictions are considered depending on the importance of the indicators (inputs and outputs) to determine the efficiency of the units. structural changes lead to the application of new decision-making units to the model. In such circumstances, determining the desired limit in order to maintain the efficiency to scale in the model or to improve the efficiency to scale of the model under consideration becomes very important. Therefore, the purpose of this article examines the effect of applying weight restrictions on data envelopment analysis models in order to maintain or improve with It is based on the scale and with the least calculations. Finally, by presenting a numerical example, the accuracy of the stated content will be shown.

Keywords: Data Envelopment Analysis, Efficiency to the Global Scale, Efficiency to the Local Scale, Sensitivity Analysis.

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1. Introduction

Data envelopment analysis is a non-parametric method based on mathematics [1], which is used to evaluate the performance of a set of decision-making units in a production technology with multiple inputs and outputs. In this technique, a parameter called efficiency size is used to evaluate the performance of decision-making units. The efficiency of a unit is a function of various factors, including the number of units, the amount of input and output of units, the number of input and output components of units, the type of production technology and the model used.

One of the important issues in data envelopment analysis (DEA) is returns to scale (RTS). Economies to scale can provide useful information regarding the optimal size of a unit. That is, whether the unit under evaluation, which is technically efficient, can improve its production by changing its size or not. In order to achieve a favorable evaluation for making appropriate management decisions, taking into account the fact that efficiency to scale is a local phenomenon for each unit under evaluation, efficiency to scale right and left [2] in coverage analysis The data was very interesting.

But some facts should be taken into consideration: 1) Proportional changes in inputs do not necessarily lead to proportional changes in outputs. 2) If the manager wants to know about the increase in one of the specific components of the output vector after making changes in the input vector, then the radial models will not be able to respond. 3) Many production technologies are not convex, so it is not possible to use the concepts defined for local scale efficiency that examines the unit scale efficiency under evaluation in a neighborhood of that point. By determining the return to scale [3] for a decision-making unit, a decision can be made regarding the expansion or limitation of that unit. To evaluate decision-making units, we must first know the relationship between the changes in their inputs and outputs. The ratio of these changes is called efficiency to scale [4]. that the yield to the local scale in a decision-making unit is four types: increasing, decreasing, fixed and variable. On the other hand, returns to scale are not always estimated in desirable outputs, so the concept of loss to scale is brought up, which is actually return to scale with undesirable outputs. The concept of MPSS in data envelopment analysis is that the decision-making unit under evaluation is an MPSS if and only if we reduce the inputs by α and increase the outputs by β and it is still in the production possibility set. and have β ≥ α. In other words, if the decision-making unit under evaluation is efficient under the BCC model, then it is efficient under the similar CCR model with constant returns to scale. The common region of the efficient frontier of CCR and BCC that applies to this property is called MPSS. Also, the decision making unit $\left(X_o, Y_o\right)$ is MPSS and if and only if it has the highest optimal value of the objective function of the data envelopment analysis model among all DMUs. Therefore, according to the concept of MPSS, we can define an opposite concept as the worst productivity measure (MPSS), the decision-making unit $\left(X_o, Y_o\right)$ is WPSS if and only if the optimal value of the objective function of the pessimistic CCR model is equal to be 1. The use of efficiency to the national scale can distinguish the decision-making units that are MPSS, and finding the optimal decision-making unit represents the MPSS.

In many multifaceted production technologies, the reference units are not clear, based on which it is possible to estimate the type of yield on a unit scale, also sometimes the used production technology is non-convex (FDH production technology), which makes it possible to use the yield to The local scale means checking the condition of the desired unit in a neighborhood of that unit will cause problems. The concept of yield to the global scale [5] indicates the direction to reach MPSS faster, which cannot be defined by the local production function. Local scale efficiency is defined based on the concept of scale elasticity, so it will be difficult to find a non-radial direction to reach MPSS. The managerial meaning of returns to the MPSS scale will be difficult. The management meaning of returns to local and national scale are different. Efficiency to local scale indicates the type of change of unit scale in which productivity should be available. While efficiency to the global scale determines the type of change of scale in which the maximum global productivity can be obtained [6].

2. Previous methods

Suppose the observed DMUs are $\left(X_j, Y_j\right), j=1,\dots,n$. In this case $X_j \in \mathbb{R}_+^m$ and $Y_j \in \mathbb{R}_+^s$ will be the input and output vectors, respectively. More specifically, consider the radial efficiency evaluation of the DMU_o output by the multiplicative VRS model. This model is expressed by $v \in \mathbb{R}_+^m$ and $u \in \mathbb{R}_+^s$ in terms of variable vectors of input and output weights, respectively.

Weight limits are additional limits on input and output weights expressed by the multiplicative model. Let's assume that k up to the homogeneous weight limits is expressed as follows:

The radial efficiency of DMU_o output is the inverse of the optimal value of η^* the following program: [7]

$$
\begin{aligned}
\eta^* = & \min v^T X_o + w \\
s.t \quad & u^T Y_o = 1 \qquad\qquad (2) \\
& v^T X_j - u^T Y_j + w \geq 0, \; j = 1,\dots,n \\
& u^T P_t - u^T Q_t \geq 0, \qquad t = 1,\dots,k \\
& y, v \geq 0, w \text{ free}
\end{aligned}
$$

From the dual model (2) above, the cover form model of the nature of the output can be obtained, which can be expressed as follows:

$$
\begin{aligned}
\eta^* = \quad & \max \eta \qquad\qquad (3) \\
s.t \quad & \sum_{j=1}^{n} \lambda_j X_j + \sum_{t=1}^{k} \pi_t P_t \leq X_o, \\
& \sum_{j=1}^{n} \lambda_j Y_j + \sum_{t=1}^{k} \pi_t Q_t \geq \eta Y_o, \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda, \pi \geq 0, \quad \eta \text{ free}
\end{aligned}
$$

which is the dual vector $\pi \in \mathbb{R}_+^k$ of weight restrictions in model (2).

You can see that the cover model (3) includes the dual parts that are generated by the weight constraints in the multiplicative model (2).

$$\left(P_t, Q_t\right), \qquad t = 1,\dots,k \qquad (4)$$

These parts, which are used in variable proportions $\pi_t \geq 0$, change and modify the DMUs in the standard VRS model, which represent the first parts of the restrictions mentioned in program (3). According to Podinoski (2004) [8], the parts mentioned in formula (4) can be interpreted as a trade-off between outputs and inputs. This indicates that the envelope program (3) can evaluate the radial efficiency of the DMU_o output in the VRS developed by the trade-off production (4).

More precisely, this developed technology can be defined as follows:

Definition 1: (Podinoski 2004) [9] VRS technology with T_{VRS-TO} trade-off generation is a set of all non-negative DMUs $(X,Y)\in\mathbb{R}_{+}^{m}\times\mathbb{R}_{+}^{s}$ which are intensity vectors $\lambda\in\mathbb{R}_{+}^{n},\pi\in\mathbb{R}_{+}^{k}$ slack vectors $d\in\mathbb{R}_{+}^{m}$ and $e\in\mathbb{R}_{+}^{s}$ exist for them so that we will have the following formula:

$$\sum_{j=1}^{n}\lambda_j X_j+\sum_{t=1}^{k}\pi_t P_t+d=X, \qquad (5a)$$

$$\sum_{j=1}^{n}\lambda_j Y_j+\sum_{t=1}^{k}\pi_t Q_t-eY, \qquad (5b)$$

$$\sum_{j=1}^{n}\lambda_j=1 \qquad (5c)$$

T_{VRS-TO} technology is a multifaceted set and therefore it is considered a convex technology. Podinovski (2015) [7]

3. The Proposed Method

In this part, using BCC models, we will examine the return to scale by the left and right return to scale method, and then we will check it by adding weight restrictions (1) to this model.

Suppose that α is proportional changes in all inputs and β is proportional changes in all outputs. We denote the set of all proportional changes for (X_o,Y_o) by $P(X_o,Y_o)$ and we have:

$$P(X_o,Y_o)=\left\{\begin{array}{l}(\alpha,\beta)\,|\,\alpha X_o\le\lambda X\\ \&\beta Y_o\ge\lambda Y\\ \&1\lambda=1\&\ \lambda\ge0\end{array}\right\} \qquad (6)$$

Theorem 1: If in the following model

$$\min\frac{\alpha}{\beta} \qquad (7)$$

$$\begin{aligned}s.t\quad &\sum_{j=1}^{n}\lambda_j X_{ij}\le\alpha X_{io}, && i=1,\dots,m\\ &\sum_{j=1}^{n}\lambda_j Y_{rj}\ge\beta Y_{ro}, && r=1,\dots,s\\ &\sum_{j=1}^{n}\lambda_j=1\\ &\lambda_j\ge0,\ j=1,\dots,n, && \alpha,\beta\ge0\end{aligned}$$

$(\lambda^*,\alpha^*,\beta^*)$ is the optimal solution, then:

A) If $\alpha^*=\beta^*=1$, then it is possible to produce (X_o,Y_o) constant returns to scale.

b) If $\alpha^*<\beta^*<1$, then it is possible to produce (X_o,Y_o) decreasing returns to scale.

c) If $1<\alpha^*<\beta^*$, then it is possible to produce (X_o,Y_o) increasing returns to scale.

Now, if we add the weight restrictions (1) to the above model, we will have the following model by rewriting it:

$$\min\frac{\alpha}{\beta} \qquad (8)$$

$$\begin{aligned}s.t\quad &\sum_{j=1}^{n}\lambda_j X_{ij}+\sum_{t=1}^{k}\pi_t P_t\le\alpha X_{io},\ i=1,\dots,m,t=1,\dots,k\\ &\sum_{j=1}^{n}\lambda_j Y_{rj}+\sum_{t=1}^{k}\pi_t Q_t\ge\beta Y_{ro},\ r=1,\dots,s,t=1,\dots,k\\ &\sum_{j=1}^{n}\lambda_j=1\\ &\lambda_j,\pi_t\ge0,\qquad \alpha,\beta\ge0\end{aligned}$$

The addition of weight restrictions causes the removal of a group of structural hyperplanes and sometimes leads to the application of new decision-making units to the model, so in this situation, determining the desired limit in order to maintain the efficiency to scale in the model or to improve the efficiency to scale The studied model is very important.

Therefore, in the model without weight restrictions, if the DMU is efficient, it may become inefficient with the addition of weight restrictions, and we do not calculate the return to scale for that, and this is a weakness of the previous methods.

4. Numerical example:

In this section, we present the contents stated in the previous section in the form of a numerical example, for a better understanding of the subject.

Table 1 Data sets used in the example

	InPut 1	InPut 2	InPut 3	OutPut 1	OutPut 2
DMU 1	2	2	3	2	1
DMU 2	4	4	3	4	3
DMU 3	6	4	5	3	2
DMU 4	2	3	3	3	4
DMU 5	6	5	4	4	5

Table 2: Result of RTS

	Podinowski model		Yield model to right and left scale without weight limit				Return model to right and left scale with weight limit			
	E	RTS	α^*	β^*	E	RTS	α^*	β^*	E	RTS
DMU 1	1	IRS	1.5	1.5	1	IRS	1.25	2.5	0.5	U.D
DMU 2	1	CRS	1	1	1	CRS	1	1	1	CRS
DMU 3	0.5	U.D	0.6	0.8	0.75	U.D	0.6	1.3	0.45	U.D
DMU 4	1	CRS	1	1	1	CRS	1	1	1	CRS
DMU 5	1	DRS	0.75	0.77	0.96	DRS	0.75	0.7	0.96	DRS

The set of data mentioned in Table (1) shows 5 DMU items that are evaluated on 3 inputs and 2 outputs. Assume that v_1, v_2, v_3 are input weights and u_1, u_2 are output weights. Consider evaluating the radial output efficiency of five DMUs using the VRS model under the additional weight constraints below:

$$v_1 - v_2 \geq 0, \qquad (9)$$

$$-v_1 + 2v_2 \geq 0,$$

$$-2u_1 + 3u_2 \geq 0,$$

$$v_1 - 2u_2 \geq 0,$$

Table 2 includes the radial output efficiency of DMUs in different models with weight limit and without weight limit (9) as well as α^*, β^* values and their RTS characteristics.

The radial output efficiency of each DMU can be evaluated by solving the multiplicative model (2) or the dual coverage model (3) and the efficiency model to the left and right scale (7). Calculations according to table (2) show that four out of five DMUs have radial output efficiency and DMU 3 is introduced as an inefficient unit.

Now, if we add weight constraints (9) to model (7) or solve model (8) with weight constraints (9), we can see that in addition to DMU 3, DMU 1 also becomes inefficient and the estimation The efficiency is not done to its scale.

For example, in DMU 4, in the model of returns to the right and left scales without weight restrictions and with weight restrictions (9), the return value to the right scale $\left(\beta^* = 1\right)$ and the return value to the left scale $\left(\alpha^* = 1\right)$ is equal to one and according to theorem (1) this DMU has constant returns to scale (CRS), which is the same as the result obtained from the Podinowski model.

According to the efficiency values of DMUs and comparing their RTS in three different models in table (2), we find that in the Podinowski model and the model of efficiency to scale on the right and left without weight restrictions (9), the efficiency to scale is the same in both models. Is.

But by adding weight constraints (9) to the efficiency model to the right and left scales with weight constraints (9), efficiency changes in some DMUs.

For example, the efficiency of the DMU 1 image has been changed, and in addition to DMU 3, which was ineffective in all three models, the image of DMU 1 is also ineffective in this model, and we cannot determine its efficiency to scale. Therefore, by adding these weight restrictions, the image DMU 1 becomes inefficient and the return to scale is not determined for it, and the condition of checking whether the DMUs remain efficient or not should be done, which is not done, and this is a weakness of the Podinowski model.

5. Conclusion

One of the important issues in data envelopment analysis (DEA) is returns to scale (RTS) and maximum productivity size (MPSS) of decision-making units and its estimation. Economies to scale can provide useful information regarding the optimal size of a unit. That is, whether the unit under evaluation, which is technically efficient, can improve its production by changing its size or not. In order to achieve a favorable evaluation for making appropriate management decisions, taking into account the fact that efficiency to scale is considered a local phenomenon for each unit under evaluation, efficiency to scale to the right and left in data coverage analysis, It got a lot of attention.

In the article of Podinovski (2017), a number of the most important methods to estimate RTS by adding weight restrictions were examined. Each DMU is Therefore, RTS detection for a specific DMU without considering and applying a collective sensitivity analysis in the neighborhood of that DMU is meaningless.

In this article, we investigated the estimation of RTS by the efficiency method to the left and right scale by applying weight restrictions, and by comparing it with the methods presented in the article of Podinovski (2017), we reached the conclusion that, by applying weight restrictions to The discussed model of efficiency and efficiency to the scale of some DMUs has not changed compared to the Podinovsky model (2017), but the efficiency of some DMUs has changed and they become inefficient compared to the model without weight restrictions and the Podinovsky model (2017) and in this case Estimating efficiency to scale is impossible and this is a weakness that can be found in Podinovski's (2017) model. Because the condition of whether the decision-making unit will remain efficient by applying the weight limit has not been fulfilled and the inefficiency of some DMUs leads to non-recognition. Its RTS becomes.

Now, in order to solve this problem, we suggest that weight restrictions be introduced in such a way that the estimation of return to scale can be done with any usual method, the RTS does not change, while the restrictions applied by Podinovski do not have this property.

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