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Research Article

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Capacity utilization in data envelopment analysis with integer data

Balal karimi[a] ,Esmaeil Khorram[b*]

(a) *Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran*

(b) *Department of Mathematics and Computer Science, Amirkabir University of Technology, Hafez Ave.; Tehran, Iran.*

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Abstract

In the process of production, some of the inputs are fixed and cannot easily be changed, such as work hours of workers and hours of administrative or work, these types of inputs are fixed and others are variables. In this paper, by considering some inputs or outputs which are limited, their amounts must be integrated; this concept for integer data is extended. We show the importance of subject by bringing a real example of 25 branches of a bank.

Keyword: Data envelopment analysis, Integer data, Capacity Utilization.

1. Introduction

Capacity Utilization deals with a situation where some of the input values are fixed and cannot simply be changed, while others are flexibly changed. Therefore, we classify inputs into fixed and variable inputs and accordingly evaluate the effect of productivity of variable inputs. The consolidation of variable inputs, whether increases or not the efficiency value of evaluation decision making unit (DMU). Researcher such as Fare et al. [7] and Coilli et al. [3] have developed capacity utilization within the framework of data envelopment analysis (DEA). Tone [4] expanded SBM model for evaluation capacity utilization.

[*] Corresponding author *:* eskhor@aut.ac.ir

1. Introduction

Recently Sahoo and Tone [12] have studied capacity utilization for Indian banks.

According to the standard *DEA* models, inputs and/or outputs have real values. But in some cases, some inputs and outputs are limited to integer values. First Lozano and Villa [10, 11] emphasized the difference between two cases, and provided a Mix Integer programming (*MIP*) model to evaluate the efficiency with integer inputs and outputs. After that, Kuosmanen and Kazemi Matin [8, 9] by developing standard axioms in *DEA* [1] presented integer production possibility set (PPS) and according to it, proposed models to evaluate efficiency with integer inputs and outputs. Many scholars have worked on the *DEA* with the integer data. [2, 6].

Banks as financial institutions in each country play an important role in the economy. In this paper, we measure capacity utilization of branches of a commercial bank in Iran. But because some of the inputs and outputs of branches are limited to integer value, we expand the concept of capacity utilization with integer inputs and outputs for this commercial bank.

The remainder of this paper is organized as follows. In the next section, the concept of capacity utilization in *DEA* will be introduced. In Third section, we provide models to evaluate efficiency with integer inputs and outputs, then by using it, propose a method for capacity utilization of integer data. We illustrate importance of subject by an example of real world contain 25 branches of a commercial bank in Iran. Section 5 includes conclusion and further work.

2. Technical capacity utilization

Suppose that have n DMUs as $\{(x_j, y_j) | j = 1, \dots, n\}$, that $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})^T$ are non-negative input and output vectors unit j, respectively. Also $X = [x_1, \dots, x_n]^T$ and $Y = [y_1, \dots, y_n]^T$ are matrixes of $m \times n$ inputs and $s \times n$ outputs, respectively. Also inputs are divided into two categories. Actually, fix and variable inputs vectors are indicating by $x_j^F = (x_{1j}^F, \dots, x_{fj}^F) \in R^f$ and $x_j^V = (x_{1j}^V, \dots, x_{Vj}^V) \in R^V$, respectively and $R^m = R^f \cup R^V$.

The efficiency value to evaluate *DMUo* by using output oriented *SBM* model (*SBM-O*) is as follow:

Model 1

$$\phi^0 = \max_{\lambda, s^+} (1 + \frac{1}{s} \sum_{r=1}^{s} \frac{s_r^+}{y_{ro}})$$

$$S.t$$

$$x_{i0}^F \geq x_{i1}^F \lambda_1 + \dots + x_{in}^F \lambda_n (i = 1, \dots, f)$$

$$x_{i0}^V \geq x_{i1}^V \lambda_1 + \dots + x_{in}^V \lambda_n (i = 1, \dots, \upsilon)$$

$$y_{r0} = y_{r1} \lambda_1 + \dots + y_n \lambda_n - s_r^+ (r = 1, \dots, s)$$

$$1 = \lambda_1 + \dots + \lambda_n, \lambda_j \geq 0 (\forall j), s_r^+ \geq 0 (\forall r)$$

Where λj are structural variables to use convex combination of observed *DMUs*. *V* and

F indexes use for fix and variables inputs, respectively. Also, slack variables s_r^+ *(r=1,…,s)* are used for increasing in outputs and objective function to maximize it. Now, if we eliminate the constraints in variable inputs, the model 1 is converted as follow:

Model 2

$$\phi^0 = \max_{\lambda, s^+} (1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}})$$

$$S.t$$

$$x_{i0}^F \geq x_{i1}^F \lambda_1 + \ldots + x_{in}^F \lambda_n (i = 1, \ldots, f)$$

$$y_{r0} = y_{r1}\lambda_1 + \ldots + y_n \lambda_n - s_r^+ (r = 1, \ldots, s)$$

$$1 = \lambda_1 + \ldots + \lambda_n, \lambda_j \geq 0 (\forall j), s_r^+ \geq 0 (\forall r)$$

The difference between model 1 and model 2 is in the constraint variable inputs, which in the model 2 is eliminated the corresponding constraint variable inputs, and this work is for measuring capacity utilization. Indeed, suppose that ϕ^* and ϕ_F^* are optimal value of model 1 and 2 respectively, then measure of capacity utilization is defined by the following equation [4]:

$$k_o^* = \frac{\phi^*}{\phi_F^*} (\leq 1)$$

So if $k_o^* < 1$ and/or $\phi_F^* > \phi^*$, this indicates that some boundary restrictions in model 1 for variable inputs are active and average expansion rate can be increased by removing these boundaries for variable inputs. In this section, we introduced technical capacity utilization. In the following section, extend this concept for integer data.

3. Technical capacity utilization with integer data

Suppose that have a set of *n* DMUs, which each *DMU* contains *m* inputs and *s* outputs. In this section, suppose that inputs and outputs all are integer, and extend the concept of capacity utilization for it. But for hybrid integer and real data concept can be easily discussed.

In the following, we extend *SBM* model [13] for integer inputs and outputs. Indeed, *SBM* model for integer inputs and outputs will be presented as follows:

Model 3

$$\text{Max } \rho^* = 1 + \frac{1}{s}(\sum_r S_r^+ / y_{r0})$$

$$\text{s.t} \quad \sum_{j=1}^{n} \lambda_j \boldsymbol{x_{ij}} + S_i^- = x_{io} \quad i \in F$$

$$\sum_{j=1}^{n} \lambda_j \boldsymbol{x_{ij}} + S_i^- = x_{io} \quad i \in V$$

$$\sum_{j=1}^{n} \lambda_j \boldsymbol{y_{rj}} - S_r^+ = y_{ro}$$

$$\boldsymbol{\sum_{j=1}^{n} \lambda_j = 1}$$

$$\lambda_j \geq \text{o} \quad \text{j} = 1, \ldots, \text{n}$$

$$(S^-, S^+) \in Z_+^{m+s}$$

Where λ_j (j=1,…, n) are structural variables for convex combination of observed *DMUs*, and S_i^- , S_r^- are integer slack variables for decreasing inputs and increasing outputs, respectively. *V* and *F* indexes use for fix and variables inputs, respectively. Constraint $(S^-, S^+) \in Z_+^{m+s}$ use to grantee integer non-negative value of slack variables.

In the objective function the aim is to obtain the maximum increasing in outputs. If $(\lambda_j^*, S^{-*}, S^{+*})$ is optimal value for model 3, then target point for *DMUo* is as follow:

$x_{ij}{}^* = x_{ij} - S^{-*}{}_i$

$y_{rj}{}^* = y_{rj} + S^{+*}{}_r$

Where is an integer points. Now according to previous section, if eliminate constraints related to input variables, we will convert the model 3 as follow:

Model 4

$$\text{Max}\rho_{\text{F}}^* = 1 + \frac{1}{s}\left(\sum_r {S_r^+}/{y_{r0}}\right)$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \boldsymbol{x_{ij}} + S_i^- = x_{io} \quad i \in F \\ & \sum_{j=1}^{n} \lambda_j \boldsymbol{y_{rj}} - S_r^+ = y_{ro} \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq \text{o} \quad \text{j} = 1, \dots, \text{n} \\ & (S^-, S^+) \in Z_+^{m+s} \end{aligned}$$

The difference between model 4 and model 3 is in the constraint corresponding variable inputs, where in model 4 eliminated these constraints. Suppose that ρ^* and ρ_{F}^* are optimal values for model 3 and 4 respectively, then capacity utilization for integer data could be measured as follow:

$$K_o^* = \frac{\rho^*}{\rho_{\text{F}}^*} \ (\leq 1)$$

If $K_o^* < 1$, this indicates increasing of the efficiency value after eliminating constraints variable inputs.

In this section, capacity utilization for integer inputs and outputs was extended. In the next section, we use this approach for data in real world.

4. Numerical example

In this section, in order to illustrate importance of this subject, use the approach presented in above section for real world. Banks as financial institutions play an important role in each country. Knowing the current situation, a bank has an important role in economic development. Many researchers evaluated the banks by using *DEA* technique. In this section, according to capacity utilization method and its importance, measure capacity utilization branches of one commercial banks in Iran.

In this section, we use 25 branches of a bank with 3 inputs and 4 outputs to show the importance of the subject. Three inputs of bank include the number of personnel, number of Pos, and branch location. The outputs contain a number of documents, number of facilities, the claims, and the number of cheques assigned to other banks. As you know, only the location of the branch is a real data, but the rest of inputs and outputs are limited to number and are integer. Information about the inputs and outputs of the 25 branches are shown in Table 1.

Among these inputs, the number of personnel as input variables and two other inputs, it means that, number of Pos, and the location of branches are considered as fixed inputs. Now capacity utilization for number personnel of these 25 branches is calculated. But as we explained, data of inputs and outputs were limited that in order to be integer values. Therefore, we use models 3 and 4 to calculate

Table1: Information of inputs and outputs for 25 bank branch

Branches	number of personnel	number of Pos	branch location	number of documents	number of facilities	claims	cheques assigned to other banks
1	9	3	2	101604	13	0	390
2	8	3	4.2	112787	12	335	33
3	6	3	3.2	53239	3	9	19
4	9	6	4.4	100245	82	55	208
5	8	1	1.8	75715	117	178	113
6	9	4	4.8	82234	10	72	136
7	13	2	4.4	211685	704	59	144
8	12	3	4	128529	82	123	206
9	7	3	2.8	71017	66	9	57
10	13	3	3.2	165115	17	185	495
11	9	2	3.4	92818	12	102	52
12	7	2	2.6	58878	22	13	117
13	11	3	2.2	114460	103	336	168
14	4	2	2.2	33861	10	0	38
15	14	2	3.2	159212	33	15	90
16	6	3	3.4	59124	30	1	66
17	9	3	3.2	91771	28	283	106
18	9	2	4.2	125847	101	1	86
19	7	3	4.4	85482	15	12	198
20	8	2	5	93932	42	24	211
21	7	3	4.4	83942	8	63	0
22	9	3	3.8	111050	26	669	301
23	39	2	4.4	104317	80	17	284
24	8	2	4.6	83728	4	70	80
25	14	4	3.4	177630	41	35	282

capacity utilization of personnel of the 25 branches. We use *Lingo 11* software to run models 3 and 4. The results of run models and measured capacity utilization are shown in Table 2 below.

Table2: Technical capacity utilization with integer data

Branch of Bank	Optimal value of model 3 (ρ^*)	Optimal value of model 4 (ρ_F^*)	$K_o^* = \frac{\rho^*}{\rho_F^*}$ (≤ 1)
1	1	1	1
2	1	8.76	0.11
3	15.13	40.93	0.36
4	3.33	3.62	0.91
5	1	1	1
6	8.85	18.14	0.48
7	1	1	1
8	2.15	2.15	1
9	8.39	14.41	0.58
10	1	1	1
11	8.34	10.97	0.76
12	6.61	9.05	0.73
13	1	1	1
14	1.5	1.5	1
15	1	1	1
16	47.49	47.49	1
17	2.86	3.51	0.81
18	32.17	81.90	0.39
19	1	15.07	0.06
20	1	5.28	0.18
21	1	1	1
22	1	1	1
23	3.94	4.03	0.97
24	18.75	44.07	0.42
25	1	1	1

In Table 2, columns 2 and 3, give optimal value of model 3 and 4 for 25 branches of a bank. In the fourth column, amounts of capacity utilization are shown for the 25 branches. As you can see from Table 2, from 25 branches only 12 branches amounts of capacity utilization is equal 1, and the other branches amount of capacity utilization is less than 1. The more amount of capacity utilization is lower, it indicates low capacity utilization of number personnel of this branch. The lowest capacity utilization for branches, is related to branch 19, which is equal 0.06.

5. Conclusion

In this paper, we measured capacity utilization of a bank branches by using *DEA*. By considering that some of the inputs and outputs branches of bank are limited to the integer values, we expanded the capacity utilization with integer data. In this case, we have provided two integer models and using them to measure capacity utilization for integer inputs and outputs for input variables. Finally, by using this method, we analyzed the capacity utilization of 25 branches of a commercial bank.

In real world, due to the fact that, data can be limited to a specific type, we can expand capacity utilization. For example, we can use negative data, interval data, and/or undesirable outputs, which can be considered as future work.

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