Presenting a New Model for Bank’s Supply Chain Performance Evaluating with DEA Solution Approach

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Abstract

Data Envelopment Analysis (DEA) is a method for measuring the efficiency of peer decision making units (DMUs) with multiple inputs and outputs. The traditional DEA treats decision making units under evaluation as black boxes and calculates their efficiencies with first inputs and last outputs. This carries the notion of missing some intermediate measures in the process of changing the inputs to outputs of DMU and as a result the effect of these measures in the process of performance evaluation is not considered. Recently, some models are created in DEA which can evaluate the system in multi stages and consider the relations between the systems. The objective of this paper is to investigate efficiency decomposition in a three-stage process that has a two independent parallel stages linking with a one final stage. This three stage processes calculate the efficiency of organization with considering intermediate constraints. Finally, we illustrate the proposed method with numerical example.

Keyword: Data Envelopment Analysis (DEA), efficiency, Intermediate measure, Multi stage.

1. Introduction

Nowadays, the performance evaluation discussion of organizational units and consequently its impression on the overall performance of the institution is very challenging and important. In fact, to achieve its goal, including customer satisfaction and maximizing profits, the organizations should assess

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the performance of their unites based on appropriate criteria. Namely before the beginning of the improving process, they must have clear picture of their structure because getting accurate and actual results of operation can provide relevant context to decide. DEA is a nonparametric method that includes techniques for evaluating an organization’s efficiency or productivity and using it can create the possibility for its management which in order to reach ideals, the right attitude on the performance of collection can adjusts collection’s system processes and units, applying to useful one, also, can be as a pattern for all unites. Thus, the basic and classic models of data envelopment analysis came to the existence to calculate the efficiency of decision making unites considering the inputs and outputs. In recent years, due to more complexity of systems and increasing the size of organizations processes, the efficiency of these classical models declined and required a more detailed analysis of the decision making unites. Perhaps the more important reason for the decline in the effectiveness of these methods for larger systems is that the classical models do not consider internal processes of systems and they are to be accounted as a “Black box”, whereas in the multi-stage data envelopment analysis which are a special case of that of network\(^1\), relevance and linkage between internal processes is considered for determining the efficiency of the system.

Now, due to the fact that this study’s structure of proposed supply chain consists of 3 sub DMUs and more than one stages, we decided, for this complicated structure performance assessment that can concurrently take the impact of these intermediate indexes into account, use multi-stage data envelopment analysis which is a special state of network data envelopment analysis.

### 2. Multi-stage Data Envelopment Analysis

In the performance assessment process, using DEA, new comments became propounded and systematic one has changed. The processes have been broken to allow the impact of intermediate indicators on institution’s performance to be studied. As to this study, the applied DEA models can be divided into two categories:

#### A) Close system

These are the process in which output intermediate variables enter into the next stage without any change, in the other words; output intermediate variables in any stage are the sole inputs for the next. The simplest example of the system can be observed in fig.1. In this figure, a consecutive system has shown that the whole process consists of two connected series sub-systems. As is evident from the fig.1, all outputs of the first subsystem enter into the second as input.

\(^1\)Network Data Envelopment Analysis (NDEA)
In this line, as an example, Seiford and Zhu [11] used the structure of a two-stage network model to measure the profitability and supply ability of American banks. In the proposed model, the profitability toward entry personnel, assets, factory owner shares and output profit and revenue is considered in the first step. In the second phase, also, by the use of the inputs come from the first phase i.e. profit and income than outputs of market value, return of capital and earning per share are evaluated. Zhu [15] continued the same routine again and using data envelopment analysis to appraise the economic efficiency of over 500 the world’s top companies. Somewhere or other Chilingerian and Sherman [3] offered a two-stage process, using DEA to study clinicians’ attention rate.

By dividing the processes and the systems into subsystems, it’s easier to identify inefficient sources that can be applied to remove inefficiencies. This concept of a two-step analysis has been applied in the other researches such as performance assessment of mental health care, education, information technology, purchasing and supply management and suchlike [2,8,9,11]. In all above-mentioned cases, three levels of independent performance for the first stage, second stage and whole process achieve and DEA model that calculate the values of efficiency doesn’t reflect the relationship between the parts and the whole system. Chiang Kao, Shiu- Nenwang [7], different from previous studies that had an independent look on the whole process and sub-processes, with the use of the chain relationship between subsystems showed that there is a relationship between efficiency in the whole collection and subsystems . The efficiency is calculated, using DEA is more significant than that is done independently. Kao and Hwang [7] have applied this model for insurance companies in Taiwan and compared the results with the independent approach.

**B) Open system**

The process in which the output intermediate variables in one stage is as a part of the input in the next, that is, except to the first phase, the other phases have another inputs in addition to intermediate variables. Furthermore, all output values of a special phase are not essentially as inputs for others and a part of that can be expelled from the system. See fig.2 as an example. In this process, a part of the first subsystem outputs is emitted from the system or in somewhere else, a part of the second inputs enter into the system from outside and it means that the system works as an open system.
There are many examples in industries in which a part of outputs in a special phase may be available for consumers as products or services and the rest of them are imported to next phase in the process for fulfillment. The models used in this area are mostly in the network DEA mold. In these systems, each phase act as an open system and is receptive of input from outside and it possibility interacts with the last system in addition to outside ones. Goolani and Hackman [6] proposed an efficiency assessment model for systems, composing of two associated sub-system. Each of the sub-systems uses separate resources and produces outputs. E.g. these resources can be as labor or capital. The proposed network DEA model by Goolani and Hackman [6] enables to calculate the total cumulative efficiency coupled with the efficiency value of each system [9]. Castelli et al discussed over two-stage and two-layer DMUs [1]. Far and Whittaker with a network approach and Far, Grospof, Tone and Tsutsui with the approach of covariate based network DEA, are another samples [4,5,12,13].

3. Chen and Zhu DEA model

Consider the following BBC model, offering by Banker et al:

$$\theta^* = \min \theta$$

$$s.t. \sum_{j=1}^{n} \pi_j x_{ij} \leq \theta x_{io} , \quad i = 1,2,\ldots,m$$

$$\sum_{j=1}^{n} \pi_j y_{rj} \geq y_{ro} , \quad r = 1,2,\ldots,s$$

$$\sum_{j=1}^{n} \pi_j = 1$$

$$\pi_j \geq 0 , \quad j = 1,2,\ldots,n$$

In the above model, $x_{io}$ is ai-th input and $y_{ro}$ is r-th output of j-th DMU under estimation. If $\theta^*=1$, the j-th observation locates on the efficiency boundary. Otherwise, if $\theta^*<1$, then the j-th DMU is inefficient. Since the target of model (1) is to diminish the inputs and keep the outputs unchanged in a special level, it’s so-called input-oriented model.
Similarly, output-oriented model can also be found. The aim in this model is to increase outputs and to keep inputs fixed in the same level.

\[
\phi^* = \max \phi
\]

\[
s.t. \sum_{j=1}^{n} \pi_j x_{ij} \leq x_{io} , \quad i = 1,2, ..., m
\]

\[
\sum_{j=1}^{n} \pi_j y_{rj} \geq \phi y_{ro} , \quad r = 1,2, ..., s
\]

\[
\sum_{j=1}^{n} \pi_j = 1
\]

\[
\pi_j \geq 0 , \quad j = 1,2, ..., n
\]

Both model (1) and (2) consequence the same efficiency boundary since \( \theta^* = 1 \) if and only if \( \phi^* = 1 \).

Figure 3 shows a general two-step process that employs the first phase’s inputs \((i = 1,2,3,...,m)x_i\) to generate the outputs of \((d = 1,2,3,...,D)z_d\). These \(z_d\) are used as inputs of the second phase to produce the outputs of \((r = 1,2,3,...,s)y_r\).

![Fig.3 : intermediate indexes.](image)

According to supply chain structure of Wang et al in the above figure, fixed assets, the number of employees, investment, and information technology were considered as input of the first stage [14]. In the second phase, also, they estimate the performance of banks, using the profitability and loan’s deductions in them as outputs.

In order to solve these kinds of problems and intermediate indexes to be accounted, Chen and Zhu [2] developed model (1) as follows:
Min $w_1 \alpha - w_2 \beta$

(First phase)

\[ s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \leq \alpha x_{io}, \quad i = 1,2, ..., m \]

\[ \sum_{j=1}^{n} \lambda_j z_{dj} \geq \tilde{z}_{do}, \quad d = 1,2, ..., D \]

\[ \sum_{j=1}^{n} \lambda_j = 1 \]

\[ \lambda_j \geq 0, \quad j = 1,2, ..., n \]

(Second phase)

\[ \sum_{j=1}^{n} \mu_j z_{dj} \leq \tilde{z}_{do}, \quad d = 1,2,3, ..., D \]

\[ \sum_{j=1}^{n} \mu_j y_{rj} \geq \beta y_{ro}, \quad r = 1,2, ..., s \]

\[ \sum_{j=1}^{n} \mu_j = 1 \]

\[ \mu_j \geq 0, \quad j = 1,2, ..., n \]

Where $x_{ij}$ is the i-th input and $y_{rj}$ is r-th output of j unite and $i = 1,2,3, ..., m; r = 1,2,3, ..., s; j = 1,2,3, ..., n$ and "o" is an underestimation unite. $\alpha$ and $\beta$ are also efficiency values, relating to phase 1 and 2 respectively. $w_1$ and $w_2$ are allocated weights into first and second phases that if the phases have the same importance degree, they are equal to one. Moreover, $z_{dj}$ is intermediate input, accounting as input for the first and output for the second phase. The sign "\~" indicates unknown deciding variable, it signifies they are $\tilde{z}_{do}$. Chen and Zhu model is to contemplate to calculate intermediate variables in the most optimum state with the decrease of inputs and increase of outputs simultaneously. It’s worthy of mention that according to model (3), the companies take into account as efficient ones that the efficient values in both phases are equal to one ($\alpha = \beta = 1$).

**Theorem 1:** if $\alpha^* = \beta^* = 1$ then the optimum answer must be existed so that: $\lambda^*_{jo} = \mu^*_{jo} = 1$

**Theorem 2:** if $\alpha^* = \beta^* = 1$ then $\theta^* = 1$ and $\varnothing^* = 1$ where $\theta^*$ and $\varnothing^*$ are the optimum values for model 1 and 2 in order.
4. DEA model of Ebrahimnejad and colleagues for the new process

Contemplate the an organization or a system like fig.4 which have two parallel systems, referring to as A and B each of which have to distinct input groups, calling $X^1$ and $X^2$. It’s hypothesis that two systems act as two distinct systems and there is no any relationship between them. In overall, system works such a way that all systems A and B outputs consumed as the input of other connected series system, supposedly C. Output group of systems A and B name W and V respectively. Finally, systems C, also, turn intermediate variables into Y outputs.

Fig.4: a chain of two parallel and one series subsystems

Now, take an organization with n amount of similar DMU into consideration that each of DMUs act under process in fig.4. To put DMU_j (j = 1,2,3,...,n) and $x^1_{ij}$ as i-th (i = 1,2,3,...,m) input of system A and $w_t$ as t-th(t = 1,2,3,...,k) output variable of system A and input for system C and $x^2_{ej}$ as e-th (e = 1,2,3,...,c) input of system B, $v_r$ as r-th(r = 1,2,3,...,h) output variable of system B and C, and ultimately, $y_s$ as s-th (s = 1,2,3,...,q) output of system C or the whole system, under observation, Ebrahimnejad et al developed DEA model of Chen and Zhu for the special process in order to use that for assessing the same performance processes and identifying efficient and inefficient resources.

1.4. Traditional approach in performance assessing for an especial process

A common approach toward an especial problem is to encounter with the above chain as so-called Black Box and relinquish internal processes and intermediate variables. That is, we consider the whole system as a collection of inputs and outputs instead of parallel and series systems. This method is shown in fig.5.

Fig.5: inputs and outputs indiscrete presentation (Black Box)
Though, this Black Box method doesn’t consider the interior mechanism of making decision unite, it’s very useful. Because in many cases, especially about complicated systems, if we want to consider interior processes, getting information of inefficient resources is very difficult.

2.4. The analysis of Chen and Zhu DEA model about the structure of a special process

In this section, we are going to formulate the processes of fig.4, using Chen and Zhu model. After identifying the limitations and assigning the relationship between variables based on Chen and Zhu model, ultimately we will have the following linear model:

\[
\begin{align*}
\min & \quad w_1 \theta + w_2 \theta' - w_3 \beta \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \theta x_{io}^1, \quad i = 1, 2, ..., m \\
& \quad \sum_{j=1}^{n} \lambda_j w_{tj} \geq \bar{w}_{to}, \quad t = 1, 2, ..., k \\
& \quad \sum_{j=1}^{n} \lambda_j' x_{ej}^2 \leq \theta' x_{eo}^2, \quad e = 1, 2, ..., c \\
& \quad \sum_{j=1}^{n} \pi_j w_{tj} \leq \bar{w}_{to}, \quad t = 1, 2, ..., k \\
& \quad \sum_{j=1}^{n} \pi_j v_{rj} \leq \bar{v}_{ro}, \quad r = 1, 2, ..., h \\
& \quad \sum_{j=1}^{n} \pi_j y_{sj} \geq \beta y_{so}, \quad s = 1, 2, ..., q \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \sum_{j=1}^{n} \lambda_j' = 1 \\
& \quad \sum_{j=1}^{n} \pi_j = 1 \\
\lambda_j, \lambda_j', \pi_j & \geq 0; \quad j = 1, ..., n
\end{align*}
\]
θ, θ’ and β are efficiency value of A, B and C systems respectively. With the contemplation of intermediate variables as unknown ones, the model’s aim is that to decrease inputs $x_{ij}$ and $x_{e}$ and increase the outputs $y_{sj}$ at the same time, an appropriate level of intermediate variables such as $w_{tj}$ and $v_{rj}$ obtains.


Consider Chen and Zhu two-stage model to examine indirect impression of information technology on institution’s performance. This indirect examine has done such a way that information technology impacts directly on intermediate variables (bank deposits) and the effects of that on the institution performance (final output). With the attention of this issue, intermediate variables in this model are highly important and that’s the reason why they took as unknowns. It helps the indirect impression of input variables on the final output ones in a two or multi-stage system. By virtue of this issue, e.g. take intermediate variables as unknowns, some other limitations add to the model, just like Chen and Zhu two-stage model. However, in the performance assessment of a multi-stage system (for example fig.4) intermediate variables are important due to the fact that it merely creates limitations for our system and we concentrate on efficiency values of subsystems and the whole system. It’s not essential in this state to take these variables unknown and impose additional limitations onto the model. What’s important is to achieve the weights such a way that to efficiency of each subsystem and the whole system in the maximum underestimation DMU. With these interpretations, consider the limitations, relating to intermediate variables of model (4).

Limitations, relating to $w_{tj}$:
\[
\sum_{j=1}^{n} \lambda_{j} w_{tj} \geq \bar{w}_{to}, \ t = 1, \ldots, k
\]
\[
\sum_{j=1}^{n} \pi_{j} w_{tj} \leq \bar{w}_{to}, \ t = 1, \ldots, k
\]

Limitations for $v_{rj}$:
\[
\sum_{j=1}^{n} \lambda'_{j} v_{rj} \geq \bar{v}_{ro}, \ r = 1, \ldots, h
\]
\[
\sum_{j=1}^{n} \pi_{j} v_{rj} \leq \bar{v}_{ro}, \ r = 1, \ldots, h
\]

With the comparison between relationships, it’s clear-cut that the limits for intermediate variables can be combined together and decreased the number of limits from 4 to 2:
∑_{j=1}^{n} \lambda_j w_{tj} \geq ∑_{j=1}^{n} \pi_j w_{tj} , \quad t = 1, ..., k

∑_{j=1}^{n} \lambda_j' v_{rj} \geq ∑_{j=1}^{n} \pi_j v_{rj} , \quad r = 1, ..., h

The recent two limits guarantee in order that input intermediate variable to system C not to be more than output variable of system A and furthermore, input intermediate variables of system C not to be more than output system A. Now, if we want to apply the above changes on model (4) and then we reach the following linear DEA model.

\[
\min w_1 \theta + w_2 \theta' - w_3 \beta
\]

\[
s.t. \sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \theta x_{i0}^1 , \quad i = 1, 2, ..., m
\]

\[
\sum_{j=1}^{n} \lambda_j w_{tj} \geq ∑_{j=1}^{n} \pi_j w_{tj} , \quad t = 1, ..., k
\]

\[
\sum_{j=1}^{n} \lambda_j' x_{ej}^2 \leq \theta' x_{e0}^2 , \quad e = 1, ..., c
\]

\[
\sum_{j=1}^{n} \lambda_j' v_{rj} \geq ∑_{j=1}^{n} \pi_j v_{rj} , \quad r = 1, ..., h
\]

\[
\sum_{j=1}^{n} \pi_j y_{sj} \geq \beta y_{s0} , \quad s = 1, ..., q
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
\sum_{j=1}^{n} \lambda_j' = 1
\]

\[
\sum_{j=1}^{n} \pi_j = 1
\]

\[
\theta \leq 1
\]

\[
\theta' \leq 1
\]

\[
\beta \geq 1
\]

\[
\lambda_j , \lambda_j' , \pi_j \geq 0 , \quad j = 1, ..., n
\]

Model (5) is to assess performance process of fig (4). This model’s aim is that according to intermediate variables limits, with the subtraction of \(x_{ij}^1\) and \(x_{ej}^2\) inputs and augmentation of \(y_{sj}\) at the same time, the
efficiency value of A, B and C system, namely $\theta, \theta'$ and $\beta$ to be achieved for DMU$_j$. The limitations $\theta \leq 1$ and $\theta' \leq 1$, don’t allow the inputs to go over an especial level and limitation $\beta \geq 1$ warranties the outputs to don’t go down of an acceptable level. The values $\theta = \theta' = \beta = 1$ and $\lambda_{j0} = \lambda'_{j0} = \pi_{j0} = 1$ are feasible results for model (5).

5. Absorption and resource allocation process (open system) in the country banks.

Country banks act in the two overall phases (stages), in the first, the resources earmarks and afterwards in phase 2, they make money with the allocation of these resources. Here, we defined the inputs and outputs for each of these phases and its formed subsystems. To do this, we analyzed the process. Phase 1, namely resource absorption includes two distinct subsystems, calling the first source absorption (time deposit) and the second one (money on deposit for documentary credits) (fig.6). Consider that we hypothesized that these two subsystems act independently and do not have any connection together. In the second phase, independent inputs ($X^3$) with the outputs of phase 1 (as inputs) enter into the second phase. Finally, this sub-system (source allocation) invests the deposits and sources in the different shape to earn money for banks.

The significant point here is that achieved outputs in the last stage are divided into two groups: beneficial (good) and non-beneficial (bad) outputs. Beneficial outputs are that one’s which are useful for the banks and are somehow income for them. In contrast, non-beneficial outputs are ones which are maleficent for them and to be subtracted. The present connection between subsystems is shown below:

![Diagram](https://example.com/diagram6.png)

Fig.6: absorption and source allocation process (open system)
1.5. Introducing of this research understudied indexes in absorption and source allocation process.

A) Input indexes:
\( x^1 \): Payment on account profit to investors
\( x^2 \): Costumers obstructions for documentary credits and surety bonds.
\( x^3 \): The sum of prices, including doubtful arrival of claims and advertising expenditures and etc.

B) Intermediate indexes:
\( v \): Time deposit (short and long time)
\( w \): Other deposits (cash deposit for giving documentary credits and surely bonds)

C) Output indexes:
\( y^a \): The efficient outputs (sum of prices and remaining checking facilities)
\( y^b \): The non-efficient outputs (remaining non-checking facilities, including on-time and delayed and doubtful arrival claims).

6. This study’s developed DEA model, relating to open system structure.

Consider the process in fig.6. The hypothesis here is that parallel systems work separately and don’t connect together and moreover, all output intermediate variables of two parallel sub-systems plus \( x^3 \) as inputs are under authority of third subsystem. Since obtained outputs in this phase divided into efficient (good) and non-efficient (bad) outputs, to decrease the impact of non-efficient outputs, firstly output variable \( y^b \) is multiplied to -1, and then we add the value of \( t_f \) to all non-efficient outputs to turn it into positive such a way that the following relationship establishes.

\[
y_{fj}^{-b} = -y_{fj}^b + t_f > 0
\]

Plus, \( t_f \) achieves from the following relationship:

\[
t_f = \text{Max}\{y_{fj}^b\} + 1
\]

Since to create a mathematical model, it’s obligatory to assign the relationship between variables, aim or aims and identifying the present limits in the problem, we’re going to formulate the process fig. 6, using Ebrahimnejhad and colleague’s model.
$\text{min } 0.5 \theta + 0.25 \theta' - 0.25 \beta$

s.t. $\sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \theta x_{i0}^1$, \hspace{1em} i = 1, \ldots, m$

$\sum_{j=1}^{n} \lambda_j w_{tj} \geq \sum_{j=1}^{n} \pi_j w_{tj}$, \hspace{1em} t = 1, \ldots, k

$\sum_{j=1}^{n} \lambda_j' x_{ej}^2 \leq \theta' x_{eo}^2$, \hspace{1em} e = 1, \ldots, c

$\sum_{j=1}^{n} \pi_j v_{rj} \geq \sum_{j=1}^{n} \lambda_j v_{rj}$, \hspace{1em} r = 1, \ldots, h

$\sum_{j=1}^{n} \pi_j x_{ij}^3 \leq x_{i0}^3$, \hspace{1em} r = 1, \ldots, h

$\sum_{j=1}^{n} \pi_j y_{sj} \geq \beta y_{so}$, \hspace{1em} s = 1, \ldots, q

$\sum_{j=1}^{n} \pi_j y_{fj}^{-b} \geq \beta y_{fo}^{-b}$, \hspace{1em} f = 1, \ldots, u

$\sum_{j=1}^{n} \lambda_j = 1$

$\sum_{j=1}^{n} \lambda_j' = 1$

$\sum_{j=1}^{n} \pi_j = 1$

$\theta \leq 1$

$\theta' \leq 1$

$\beta \geq 1$

$\lambda_j, \lambda_j', \pi_j \geq 0$, \hspace{1em} j = 1, \ldots, n$

We considered target function coefficient according to system analyzer as follows:

$w_1 = 0.5$, \hspace{1em} $w_2 = 0.25$, \hspace{1em} $w_3 = 0.25$

$\theta$ is efficiency for system 1 and $\theta'$ for the second and $\beta$ for the third one. Now, we are going to use new offered data in the next section and developed model (6), and studied performance of 15 banks in our country in order to minimizing the inputs and non-efficient outputs and also maximizing efficient outputs and identifying efficient and non-efficient subsystems in the different banks.
7. Data table

Statistical society of research are the most important banks in our country, albeit some of the banks due to numerous reasons such as being new founded, lack of availability to data and suchlike, omitted of it. Each of these banks has especial properties like documentary credit, Payment on account profit to investors and etc., that their efficiency only can be logically assessed that homogeneous unites are to be compared. Obtained data in these banks are the average (per capita) data per human resource in banks, collecting 2012 and presented as tables (1), (2) and (3) for each subsystem.

Table1. Inputs and outputs variables values of the first subsystem (unite: per million Rials).

<table>
<thead>
<tr>
<th>Banks</th>
<th>Input variables</th>
<th>Output variables (Intermediate)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x^1$</td>
<td>$y$</td>
</tr>
<tr>
<td>Eghtesad-E-Novin</td>
<td>5704</td>
<td>47996</td>
</tr>
<tr>
<td>Parsian</td>
<td>7302</td>
<td>48322</td>
</tr>
<tr>
<td>Pasargad</td>
<td>5816</td>
<td>44439</td>
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<tr>
<td>Tat</td>
<td>1368</td>
<td>21341</td>
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<tr>
<td>Tejarat</td>
<td>959</td>
<td>10840</td>
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<td>Refah-E-Kargaran</td>
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<td>4892</td>
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<td>Melli</td>
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</table>
Table 2. Inputs and outputs variables values of the second subsystem (unite: per million Rials).

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<th>Banks</th>
<th>Input variables</th>
<th>Output variables (Intermediate)</th>
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</tr>
<tr>
<td>Shahr</td>
<td>3051</td>
<td>507</td>
</tr>
<tr>
<td>Saderat-E-Iran</td>
<td>2805</td>
<td>933</td>
</tr>
<tr>
<td>Karafarin</td>
<td>15449</td>
<td>1682</td>
</tr>
<tr>
<td>Gardeshgari</td>
<td>22159</td>
<td>3867</td>
</tr>
<tr>
<td>Melli</td>
<td>2405</td>
<td>799</td>
</tr>
</tbody>
</table>

Table 3. Inputs and outputs variables values of the third subsystem (unite: per million Rials).

<table>
<thead>
<tr>
<th>Banks</th>
<th>Input variables</th>
<th>Output variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x^3$</td>
<td>$y^g$</td>
</tr>
<tr>
<td>Eghtesad-E-Novin</td>
<td>1226</td>
<td>34204</td>
</tr>
<tr>
<td>Parsian</td>
<td>1202</td>
<td>46327</td>
</tr>
<tr>
<td>Pasargad</td>
<td>963</td>
<td>46330</td>
</tr>
<tr>
<td>Tat</td>
<td>3255</td>
<td>27858</td>
</tr>
<tr>
<td>Tejarat</td>
<td>728</td>
<td>15651</td>
</tr>
<tr>
<td>Refah-E-Kargaran</td>
<td>549</td>
<td>4921</td>
</tr>
<tr>
<td>Saman</td>
<td>1674</td>
<td>33890</td>
</tr>
<tr>
<td>Sepah</td>
<td>841</td>
<td>11459</td>
</tr>
<tr>
<td>Sarmayeh</td>
<td>1780</td>
<td>23551</td>
</tr>
<tr>
<td>Sina</td>
<td>768</td>
<td>20991</td>
</tr>
<tr>
<td>Shahr</td>
<td>1111</td>
<td>15372</td>
</tr>
<tr>
<td>Saderat-E-Iran</td>
<td>905</td>
<td>8994</td>
</tr>
</tbody>
</table>
Since output variables of both first and second subsystem, including the outputs of deposit absorption (sources) on behalf of documentary credits and guaranties, are the inputs for the third subsystem or source allocation section as well, in table (3) only $x^3$ inputs and efficiency and non-efficiency outputs have been shown. Ultimately, the model estimated, using model (6) and Lingo software and each subsystem’s efficiency score have brought in table (4).

<table>
<thead>
<tr>
<th>Banks</th>
<th>$\theta$</th>
<th>$\theta'$</th>
<th>$\beta$</th>
<th>Total efficiency</th>
<th>Black box</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eghtesad-E-Novin</td>
<td>0.45593(15)</td>
<td>0.29956(11)</td>
<td>1(1)</td>
<td>0.55286(15)</td>
<td>0.75849(15)</td>
</tr>
<tr>
<td>Parsian</td>
<td>0.70176(11)</td>
<td>0.98853(2)</td>
<td>1(1)</td>
<td>0.84801(3)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Pasargad</td>
<td>0.88117(4)</td>
<td>0.24986(14)</td>
<td>1(1)</td>
<td>0.75305(9)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Tat</td>
<td>1(1)</td>
<td>0.47176(7)</td>
<td>1(1)</td>
<td>0.86794(2)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Tejarat</td>
<td>0.81937(7)</td>
<td>0.28439(13)</td>
<td>1(1)</td>
<td>0.73078(10)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Refah-E-Kargaran</td>
<td>1(1)</td>
<td>1(1)</td>
<td>1(1)</td>
<td>1(1)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Saman</td>
<td>0.63563(12)</td>
<td>0.40855(9)</td>
<td>1(1)</td>
<td>0.66995(12)</td>
<td>0.92955(13)</td>
</tr>
<tr>
<td>Sepah</td>
<td>0.78198(8)</td>
<td>0.61382(5)</td>
<td>1(1)</td>
<td>0.79445(6)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Sarmayeh</td>
<td>0.55183(14)</td>
<td>0.28837(12)</td>
<td>1(1)</td>
<td>0.59800(14)</td>
<td>0.86140(14)</td>
</tr>
<tr>
<td>Sina</td>
<td>0.57548(13)</td>
<td>0.65755(3)</td>
<td>1(1)</td>
<td>0.70212(11)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Shahr</td>
<td>0.87411(5)</td>
<td>0.49016(6)</td>
<td>1(1)</td>
<td>0.80959(5)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Saderat-E-Iran</td>
<td>0.86887(6)</td>
<td>0.43601(8)</td>
<td>1(1)</td>
<td>0.79344(7)</td>
<td>0.94227(12)</td>
</tr>
<tr>
<td>Karafarin</td>
<td>0.70398(10)</td>
<td>0.22005(15)</td>
<td>1(1)</td>
<td>0.65700(13)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Gardeshgari</td>
<td>0.96487(3)</td>
<td>0.31321(10)</td>
<td>1(1)</td>
<td>0.81073(4)</td>
<td>1(1)</td>
</tr>
<tr>
<td>Melli</td>
<td>0.76054(9)</td>
<td>0.64244(4)</td>
<td>1(1)</td>
<td>0.79088(8)</td>
<td>1(1)</td>
</tr>
</tbody>
</table>

The second to fourth columns indicate first, second and third subsystem’s efficiency performance respectively and the numbers in parenthesis are the grade of each bank regarding to the efficiency number, achieving in each relevant subsystem. To facilitate the comparison between banks, also, the fifth column has designed to be introducer of weight average value of subsystem’s efficiency (the whole
efficiency) which in this weight average, considering to the importance of sub-systems, we gave the first subsystem double weight than the others. In addition to that, the sixth column (Black Box) presents supply chain structure efficiency irrespective of intermediate indexes. As can be seen, Refah-e-karegaran bank gain number 1 in all 3 columns of performance assessment of its subsystems and therefore, it’s efficient in overall. It means that only banks which are efficient in any subsystem can be taken into account for efficient one. If we put the table into perspective, it’s scriptable that all banks are efficient in the source allocation section. In financial sources absorption (the first subsystem), Refah-e-karegaran and Tat banks were efficient 100% and Eghtesadenovin, Sarmayeh and Sina banks have the least efficiency in order. In the credit sources section (the second subsystem), it was only Refah-e-karegaran bank which has had 100% efficiency and Karafarin, Pasargad and Tejarat placed in the lowest level. Thanks to comparison between degrees of the process’s components and total efficiency, it’s easy to understand that all banks have the same degree in components and total efficiency that it indicates that total performance of collection originates from the performance of its components. In the sixth column (Black Box) which shows the efficiency of the whole supply chain structure without any consideration of intermediate indexes, it can be seen that all banks except Eghtasadnovin, Saman, Sarmayeh and Saderat-e-iran have the efficiency score equal to 1, however, the efficiency score of these banks in the relevant subsystems isn’t equal to 1 which it stipulates the importance of taking care of interior processes (indexes) to efficiency calculation of a supply chain.

8. Conclusion and the recommendations for the future

Due to the fact that traditional models act as Black Box, and don’t consider the intermediate processes, when we want to assess a complicated and chain process, they are inefficient. To cope with this problem, researchers have strived to analyze and measure complicated systems. In this order, multi-stage data covering analysis models came to the existence. This stage shows that for accurate measuring of a multi-stage system, the process must be analyzed and seen multiple subsystems connected with independent inputs and outputs or dependent (based on the special process) one. Thus, this is a way to measure the impact of unites to each other and identify efficient and non-efficient factors. A developed model is for the case that parallel subsystems don’t have any connection to each other and act separately. As a new work, this hypothesis can be broken and consider the state that there is input and output interaction between parallel subsystems. As the other research work, each of the parallel subsystem can be broken and turns into some subsystem and develops DEA model in the connection with this new structure.
References:


