



Estimation of Returns to Scale in the Presence of Undesirable (bad) Outputs in DEA When the Firm is Regulated

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Abstract

The calculation of RTS amounts to measuring a relationship between inputs and outputs in a production structure. There are many methods to measure RTS in the primal space or the dual space. One of the main approaches is using the multiplier on the convexity constraint. But returns to scale measurements in DEA models are affected by the presence of regulatory constraints. These additional constraints change the role played by the convexity constraint. In this paper discusses methods for determining returns to scale in the presence of undesirable (bad) outputs in the regulated environments.

Keywords: Returns to scale, Undesirable outputs, Regulation, Quasi-fixed inputs.

1. Introduction

Data Envelopment Analysis (DEA) is a quantitative technique, first proposed by Charns, Cooper and Rhodes (1978) [6] to measure relative efficiency for set of Decision making units (DMUs). The DEA is one of the Operations Research (OR) methods that has sparked considerable interest to itself. For a DMU, the production process is to consume the inputs to get the outputs, and the efficiency is to obtain more outputs with fewer inputs as it can. It should be noted that in the production process all the outputs is not always desirable and there may be some harmful products in the production process

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which are called undesirable outputs. An undesirable output is an undesirable result of a productive Process, whose production must be minimized. Undesirable outputs should be considered part of the production process, the failure to produce it, is almost impossible and it can be somewhat reduced. So far Different methods for calculating the efficiency in the presence of undesirable outputs are provided. For example some of them are presented here. Jahanshahloo et al (2005) [10] Proposed a non- radial DEA model in order to improve the performance of an inefficient Decision Making Unit (DMU) in the presence of undesirable outputs, and they supposed that there exist undesirable inputs, too. Rheinhard *et al*, (1999) [15] considers DEA as a multi-criteria approach, that is the undesirable output is modeled in DEA as input [11].

One of the key concepts in the theory of production is the scale of operations (RTS). It can provide benefit information about size of DMUs. RTS in DEA were introduced by Banker (1984) [1] and Banker et al (1984) [2]. Since that, there has been many attempted to evaluate RTS within the DEA context. In a more realistic environment of the DMUs, not all inputs are fully discretionary and the environment in which they operate is regulated, Ouellette et al (2012) [12] showed how to introduce these refinements of the firm's environment into the calculation of the RTS. They consequently introduced regulations as an important parts of the DMU's environment. The focus of this paper is on estimating returns to scale for DEA models when DMUs face a complex environment that includes regulation, quasi-fixed inputs and undesirable (bad) outputs. We show that RTS formula in the regulated environments with undesirable outputs is different than one used in standard case.

2. The standard approach to returns to scale characterization

There are many approaches and methodologies to measure RTS, from local to global measures, in the primal space (input–output) or the dual space (prices).

In this study we use local calculations of RTS in the primal space.

Suppose that DMUs use n variable inputs, $x = (x_1, \dots, x_n)$ to produce m outputs, (y_1, \dots, y_m) , with a technology given by a twice differentiable transformation function, $f(y, x) \leq 0$ with $\partial f(y, x)/\partial y_i \geq 0$ for $i = 1, \dots, m$ and $\partial f(y, x)/\partial x_j \leq 0$ for $j=1, \dots, n$. To measure RTS we have to measure the required change in outputs to keep the transformation function equal to zero when inputs are increased proportionally. In other words, we let the inputs be expanded proportionally along a ray by a constant factor, μ , and then we find the factor, γ , by which we multiply the outputs so that $f(\gamma y, \mu x) = 0$. For a local measure, we take the ratio of differentials:

$$RTS = \frac{d \ln \gamma}{d \ln \mu} \Big|_{\mu=\gamma=1} = - \frac{\sum_{i=1}^n f_{x_i} x_i}{\sum_{j=1}^m f_{y_j} y_j} \quad (1)$$

For details the reader is referred to Førsund and Hjalmarsson (2004) [9].

The input-oriented model to measure technical efficiency for unit h is given by the following problem:

$$\text{Min } \{\theta^h: f(y, \theta^h x) \leq 0\} \tag{2}$$

The Lagrangian of this problem is:

$$L_h^{TE} = \theta^h + \varphi f(y, \theta^h x) \tag{3}$$

where φ is the Lagrange multiplier and θ^h is Farrell's (1957) input oriented technical efficiency score. By the using first-order Taylor approximation of (3) around (x_0, y_0) and combine with (1) Ouellette et al (2012) [12] showed that RTS becomes:

$$RTS_h = \frac{\theta^h}{\theta^h - u_0} = [1 - \frac{u_0}{\theta^h}]^{-1} \tag{4}$$

If DMU is an efficient then $\theta^h = 1$ so $RTS^h = [1 - u_0^h]^{-1}$

It should be noted that Eq. (4) is the returns to scale formula derived by Førsund (1996) [8] and Førsund and Hjalmarsson (2004) [9] in the input-oriented case.

Since that piecewise linear frontiers estimated in DEA for production set, so efficient units are likely located on the vertices of a convex polyhedron. Consequently the solutions for u_0 are not unique, Førsund and Hjalmarsson (2004) [9] have proposed a procedure that consists in estimating the upper and lower bounds of u_0 by solving two linear programs.

$$u_0^{min} = \arg \min_{u_0} \left\{ \sum_{j=1}^m v_j^y y_{jh} - u_0 = 1; \sum_{i=1}^n v_i^x x_{ih} = 1; - \sum_{j=1}^m v_j^y y_{jd} + \sum_{i=1}^n v_i^x x_{id} - u_0 \geq 0, \forall d = 1, \dots, D \right\} \tag{5}$$

where D is the number of DMUs.

$$u_0^{max} = \arg \min_{u_0} \left\{ \sum_{j=1}^m v_j^y y_{jh} - u_0 = 1; \sum_{i=1}^n v_i^x x_{ih} = 1; - \sum_{j=1}^m v_j^y y_{jd} + \sum_{i=1}^n v_i^x x_{id} - u_0 \geq 0, \forall d = 1, \dots, D \right\} \tag{6}$$

For efficient DMUs this new information about u_0 permit us to measure lower and upper bound on RTS, this approach is fully characterized in Førsund and Hjalmarsson (2004).

Similar argument On the multiplicity of u_0 could be happen in our target, but this adds to the complexity of our problem we propose to address here, so it will be ignored in the rest of the paper.

Up to this point, it is assumed that all inputs and outputs can be varied at the discretion of management or other users. These may be called “discretionary variables.” “Non-discretionary variables,” not subject to management control, may also need to be considered. As Ouellette et al (2012) mentioned: *The conceptual meaning of nondiscretionary inputs contain big class of variables. Our focus here is on inputs, or factors of production, not on the general concept of non-discretionary variables. For example, a fast food outlet can be located in a highly accessible area or not and have or not a drive through facility. Both variables are non-discretionary, but as the former is definitely not a technological artefact, the latter is clearly technological. It belongs to the quasi-fixed inputs, as the firm cannot adjust the quantity used as it wishes at decision time.*

After this introduction of quasi-fixed inputs in the production process, it is necessary to adapt the definition of the technology, To go along this path, first the input vector divide in two components: The first component an n-vector of variable inputs (x) and the second component is an L-vector of quasi-fixed inputs (k).

By the using of the technical efficiency problem of Banker and Morey (1986) [3] and used by Ouellette and Vierstraete (2004) [13]. The problem of estimation Returns to Scale is:

$$\text{Min}\{ \theta^h : f(y, \theta^h x, k) \leq 0 \} \quad (7)$$

BY the method that mentioned above RTS given as:

$$RTS_h = - \frac{\sum_{l=1}^L f_{kl} k_{lh} + \sum_{i=1}^n f_{xi} x_{ih}}{\sum_{j=1}^m f_{yj} y_{jh}} = \left[1 - \frac{u_o}{\theta^h + \sum_{l=1}^L v_l^k k_{lh}} \right]^{-1}$$

2. Measuring Returns to scale in a regulated environment

The environment where firms are, generally changed by a number of constraints other than technological. One of those important factors is regulation. In other words Very few decisions in a firm are made without intersecting some regulation.

It would be desirable to introduce the regulation during the measurement efficiency of firms. Lasserre and Ouellette (1994) [11], and Ouellette and Vigeant (2001) [13], model the regulation through introducing new transformation function as

$f(y, x, k, r) = 0$, this function depends on the outputs, the variable inputs and the quasi-fixed inputs as usual and also on a Q-vector of variables, r that represent the state of the regulation. As they noted the production possibilities prohibited by the regulation. This clearly leads to a smaller opportunity set for the firm as the production possibilities are restricted.

Pierre Ouellette et al (2012) Formulate RTS in this situation as:

$$RTS^R = \left[1 - \frac{u_0 + \sum_{q=1}^Q v_q^r r_{qh}}{\theta^h + \sum_{l=1}^L v_l^k k_{lh}} \right]^{-1} \tag{8}$$

A look at the above formula, shows that u_0 multiplier does not determine alone the qualitative nature of the returns to scale the major difference being that the RTS as defined by Eq. (8) are not purely technological and must be noted that Eq.(8) is not true in CCR model.

To solve this problem, the reader is referred to [12]. Bellow example by Ouellette et al shows more details

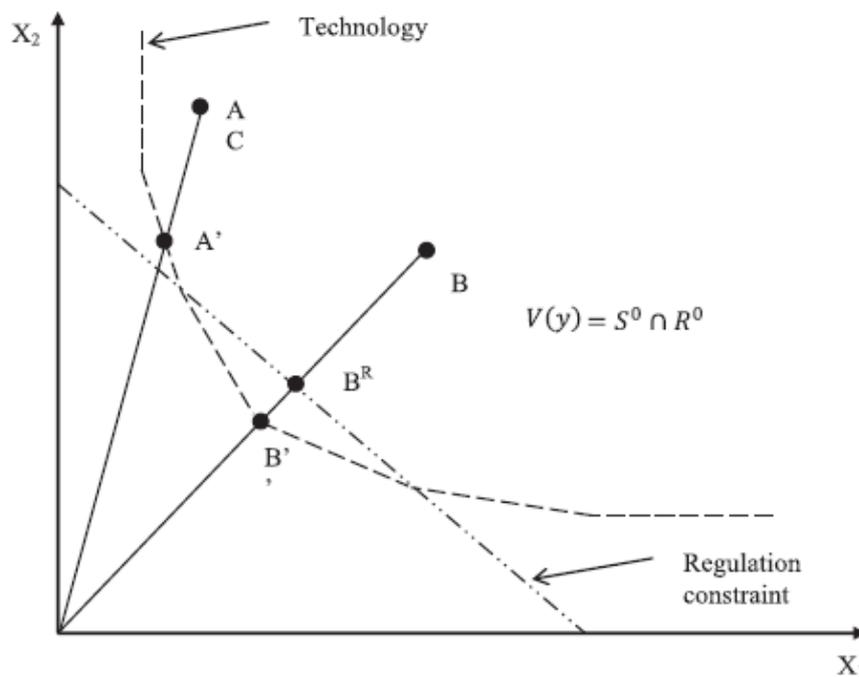


Fig1. Regulated technology

Fig1 shows that efficiency measure and RTS measure changes when the regulated constraint is binding.

3. Returns to scale for regulated environment in the presence of undesirable outputs

As mentioned, the criterion of efficiency in DEA is, increased input may reduce the efficiency while increased output may increase the efficiency.

In real cases, the circumstances are more complicated where increased output may also reduce the efficiency; this kind of outputs is undesirable outputs. For example emission of pollutant is undesirable output in production process that have harmful social and environmental dimensions. So in the measurement of efficiency the outputs should be divided into two categories as desirable and

undesirable. In this case, one should recognize technologies with more desirable outputs and fewer bad undesirable outputs relative to fewer input resources as efficient.

Our goal here is to estimation of RTS for firms that located in regulation environments and produce undesirable outputs.

For this goal first we must introduce transformation function:

$$f^R(y, g, x, k, r) \leq 0$$

Where $g = (g_1, \dots, g_t)$ is vector of undesirable outputs.

To compute efficiency for unit h the input-orient model is given by following problem:

$$\text{Min } \{\theta^h: f^R(y, g, \theta^h x, k, r) \leq 0\} \quad (9)$$

The Lagrangian of (9) is:

$$L^{TE} = \theta^h + \varphi f^R(y, g, \theta^h x, k, r) \quad (10)$$

If we add some structure to the technology, this problem has no empirical content.

In order to establish a connection between the ‘‘true’’ problem and the empirical approximation we use a first-order Taylor approximation of L^{TE} around (y^0, g^0, x^0, k^0) :

$$\begin{aligned} L^{TE} \approx & \theta^h + \varphi f^R(x^0, y^0, g^0, k^0, r^0) + \\ & \varphi \sum_{i=1}^n f_{xi}^R(x^0, y^0, g^0, k^0, r^0) (x_{ih} - x_{ih}^0) + \varphi \sum_{j=1}^m f_{yj}^R(x^0, y^0, g^0, k^0, r^0) (y_{jh} - y_{jh}^0) + \\ & \varphi \sum_{l=1}^L f_{kl}^R(x^0, y^0, g^0, k^0, r^0) (k_{lh} - k_{lh}^0) + \varphi \sum_{t=1}^T f_{gt}^R(x^0, y^0, g^0, k^0, r^0) (g_{th} - g_{th}^0) + \\ & \varphi \sum_{q=1}^Q f_{rq}^R(x^0, y^0, g^0, k^0, r^0) (r_{qh} - r_{qh}^0) \end{aligned} \quad (11)$$

Because f is unknown Eq. (11) is not implementable in practice, To estimate the unknown technology we use a DEA technique

$$\text{Min } \{\theta^h: \sum_{d=1}^D \lambda_d y_{jd} \geq y_{jh}, \quad \forall j = 1 \dots m; \sum_{d=1}^D \lambda_d x_{id} \leq \theta^h x_{ih} \quad \forall i = 1 \dots n$$

$$\sum_{l=1}^L \lambda_d k_{ld} \leq k_{lh}, \quad \forall l = 1 \dots L; \sum_{t=1}^T \lambda_d g_{td} \leq g_{th} \quad \forall t = 1 \dots T$$

$$\sum_{d=1}^D \lambda_d r_{qd} \geq r_{qh} \quad \forall q = 1, \dots, Q$$

$$\sum_{d=1}^D \lambda_d = 1; \quad \lambda_d \geq 0 \quad \forall d = 1 \dots D \quad (12)$$

The convexity constraint gives BCC model.

Lagrangian of (12) is:

$$L^{DEA} = \theta^h + \sum_{j=1}^m u_j [y_{jh} - \sum_{d=1}^D \lambda_d y_{jd}] + \sum_{i=1}^n v_i [\sum_{d=1}^D \lambda_d x_{id} - \theta^h x_{ih}]$$

$$+ \sum_{l=1}^L v_l^k [\sum_{d=1}^D \lambda_d k_{ld} - k_{lh}] + \sum_{t=1}^T v_t^r [\sum_{d=1}^D \lambda_d g_{td} - g_{th}] + \sum_{q=1}^Q v_q^r [r_{qh} - \sum_{d=1}^D \lambda_d r_{qd}] + u_o [1 - \sum_{d=1}^D \lambda_d]$$

Relation between L^{TE} and L^{DEA} can be established as follows: $f_{yj}^R \approx \frac{u_j}{\varphi}$, $f_{rq}^R \approx \frac{v_q^r}{\varphi}$ $f_{xi}^R \approx -\frac{\theta^h v_i}{\varphi}$,

$$f_{kl}^R \approx -\frac{v_l^k}{\varphi}, f_{gt}^R \approx -\frac{v_t^g}{\varphi}$$

Substituting these results in Eq1 RTS becomes:

$$RTS_h = -\frac{\sum_{i=1}^n f_{xi}^R x_i + \sum_{l=1}^L f_{kl}^R k_l + \sum_{t=1}^T f_{gt}^R g_t}{\sum_{j=1}^m f_{yj}^R y_j} = \frac{\theta^h \sum_{i=1}^n v_i x_{ih} + \sum_{l=1}^L v_l^k k_{lh} + \sum_{t=1}^T v_t^g g_{th}}{\sum_{j=1}^m u_j y_{jh}} \tag{13}$$

We can simplify Eq.13 by using dual problem associated to problem (12) that is:

$$Max \{ \sum_{j=1}^m u_j y_{jh} - \sum_{l=1}^L v_l^k k_{lh} + \sum_{q=1}^Q v_q^r r_{qh} - \sum_{t=1}^T v_t^g g_{th} + u_o;$$

$$\sum_{i=1}^n v_i x_{id} - \sum_{j=1}^m u_j y_{jd} + \sum_{l=1}^L v_l^k k_{ld} - \sum_{q=1}^Q v_q^r r_{qh} + \sum_{t=1}^T v_t^g g_{td} - u_o \geq 0$$

$$\sum_{i=1}^n v_i x_{ih} = 1 \forall d = 1, \dots, D \} \tag{14}$$

For an efficient DMU third second constraint is :

$$\sum_{i=1}^n v_i x_{id} - \sum_{j=1}^m u_j y_{jd} + \sum_{l=1}^L v_l^k k_{ld} - \sum_{q=1}^Q v_q^r r_{qh} + \sum_{t=1}^T v_t^g g_{td} - u_o = \theta^h$$

Since that RTS us given by:

$$RTS_h = [1 - \frac{u_o + \sum_{q=1}^Q v_q^r r_{qh}}{\theta^h + \sum_{l=1}^L v_l^k k_{lh} + \sum_{t=1}^T v_t^g g_{th}}]^{-1} \tag{15}$$

A look at the above formula shows that RTS measurement in the presence of undesirable outputs is very similar to RTS measurement in regulated environment, they share some common information. However, when undesirable outputs do not produce ($v_t^g = 0$) Eq. (15) is identical to Eq. (8), the technological RTS in regulated environment.

If we do not have convexity constraint ($u_0 = 0$), the measurement of RTS will not result CRS. To solve this problem, in Eq. (15) we must have $u_o = -\sum_{q=1}^Q v_q^r r_{qh}$ Using this result and substituting in the dual problem (14), and writing down the Lagrangian we obtain:

$$L^{DEA} = \sum_{j=1}^m u_j y_{jh} - \sum_{l=1}^L v_l^k k_{lh} - \sum_{t=1}^T v_t^g g_{th} - \theta^h \left(\sum_{i=1}^n v_i x_{ih} - 1 \right) \\ - \sum_{d=1}^D \lambda_d \left(\sum_{j=1}^m u_j y_{jd} - \sum_{i=1}^n v_i x_{id} - \sum_{t=1}^T v_t^g g_{td} - \sum_{l=1}^L v_l^k k_{ld} \right) \\ + \sum_{q=1}^Q v_q^r [r_{qd} - r_{qh}]$$

The primal problem can be written as follows:

$$\text{Min } \{ \theta^h : \sum_{d=1}^D \lambda_d y_{jd} \geq y_{jh}; \sum_{d=1}^D \lambda_d x_{id} \leq \theta^h x_{ih}; \sum_{d=1}^D \lambda_d k_{ld} \leq k_{lh};$$

$$\sum_{d=1}^D \lambda_d g_{td} \leq g_{th}; \sum_{d=1}^D \lambda_d (r_{qd} - r_{qh}) \geq 0; \lambda_d \geq 0, \forall d \} \quad (16)$$

For measuring CRS in regulated environment whit undesirable outputs the problem given by Eq. (16) is the one to be solved. The CRS are obtained by normalizing the regulation variables with respect to the regulation of the DMU to be evaluated.

4. Conclusion

The major contribution of this study is to define the concept of returns to scale in a regulated environment with undesirable (bad) outputs in DEA.

We have shown that the calculation of returns to scale in such a situations is different from what is expected and this difference is due to the regulation constraint.

Subsequently shown that to eliminate Convexity constraint, is not sufficient to obtain CRS. For this purpose we have to normalize the regulation variables of all the DMUs whit respect to the one of the evaluated unit in addition to omit Convexity constraint.

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