Prioritizing Contractors Selection Using DEA-R and AHP in Iranian Oil Pipelines and Telecommunication Company

N.A.Ashoori*, M.R. Mozaffari

(a)Department of Management, Science and Research Branch, Islamic Azad University, Fars, Iran
(b)Department of Mathematics, Science and Research Branch, Islamic Azad University, Fars, Iran

Received 16 September 2013, Accepted 20 November 2013

Abstract

In this article we offer a method of ranking contractors by using DEA based on analysis deficit and AHP. The process of hierarchical analysis (AHP) by providing scales from paired comparison matrix, performs the contractor’s prioritizing choice. But AHP has some problems and to solve those problems, Jahanshahloo and his colleagues presented a new model which uses DEA and standard deviation. In this article, AHP’s scales are calculated with the extension of DEA based on analysis deficit DEA-R (Ratio analysis). At the end, “Iranian Oil Pipeline and Telecommunication Company” contractors will be rank by the proposed method.

Keywords: Data Envelopment Analysis(DEA), AHP, DEA-R

1. Introduction

Nowadays in all organizations the manager’s significant duty is making decisions that determine the future of their organizations. In fact, decision making is the management essence. Managing and deciding on evaluation and choosing of contractors, which is a substantial part of doing industrial and civil projects, are too important and vital since contractors are the main parts of such projects and they are the main factors who turn the sources into the final product.

These projects require a high cost, thus in order to perform them, a suitable contractor who has the ability to finish the project under the predicted timetable and predicted resources with the assumed quality is needed. The subject is so important that most of the experts believe that sublimation and construction of all fields in our country rely on employers, business advisors and contractors; so interaction of these factors and their balanced participation prevents wasting of time and funds. It can

*Corresponding author, Email address -aliashoori57@gmail.com
also lead to a desirable performing of projects.
The usual way in choosing contractors is based on the lowest offered price, although other quantitative and qualitative indicators with various degree of importance are substantial for determining a great contractor. Every year a large number of projects in different areas are defined via companies and governmental organizations and are rendered to contractors for demonstration. An interesting point based on experience is that most of these projects are left unfinished and some of them can’t reach a desirable result. Based on researches done, the cause is directly or indirectly related to the executive contractor. This shows that in order to have a successful project, accuracy should be taken in choosing the contractor.
Dickson [5] is one of the first people who had done researches on the subject of “contractor selection.” He indicated more than 23 scales which managers use in contractor selection. After that a considerable number of related conceptual and empirical articles appeared. The conceptual ones put the emphasis on the strategic process of choosing contractors. Quality, price and timely delivery which are mentioned in these articles, are the most important factors in contractor selection process.
Siddhartha et al [13] also suggested 3 new properties including price which are project delivery time, warranty period and the contractor’s past performance score.
About competition-based profit and price, which is a competitive product in a typical competitive bidding, Khaled et al [9] state that contractors might be evaluated based on some factors such as the proposed dividend, or the opportunistic bidding behavior for changes in project and the client mismanagement. Fewer efforts have been done in the context of the connection between DEA and AHP.
Shang et al [12] and his colleagues applied DEA and AHP in accounting. They used AHP for qualitative and intangible outputs.
Jahanshahloo et al [6] invented a method of applying DEA for producing the scales in AHP. Advantages of this method are that it does not contain long computational analysis and it renders a reliable performance.
In the second section of this article we review DEA and AHP. In the third section a proposed model is presented. In section four, prioritizing of contractor selection in Iranian Oil Pipeline and Telecommunication Company is performed and there is a conclusion at the end.

2. DEA and AHP

2.1 DEA
Let us consider \( n \) DMUs, each of them characterized by \( m \) inputs and \( s \) outputs and denote a semi-positive column vector of inputs of the \( j \)-th DMU as \( X_j = (x_{1j}, x_{2j}, \ldots , x_{mj})^T \) and a semi-positive column vector of outputs as \( Y_j = (y_{1j}, y_{2j}, \ldots , y_{sj})^T \). Production possibility set (PPS) \( T \) is the set \{ \( (X, Y) \mid \) the outputs \( Y \geq 0 \) can be produced by the inputs \( X \geq 0 \) \}. The set \( T \) has the following postulates:

**Postulate 1** (Convexity). If \( (X, Y) \in T \) and \( (X', Y') \in T \), then \( (\lambda X + (1 - \lambda) X', \lambda Y + (1 - \lambda) Y') \in T \) for all \( \lambda \in [0,1] \).

**Postulate 2** (Monotonicity). If \( (X, Y) \in T \) and \( X \geq X', Y' \leq Y \), then \( (X', Y') \in T \).

**Postulate 3** (Minimum extrapolation). \( T \) is the intersection set of all \( T_j \) satisfying Postulates 1 and 2, and subject to condition that each of the observed input-output vectors \( (X_j, Y_j) \in T_j \; j = 1, 2, \ldots , n \).

Production possibility set \( T \) can be defined in algebraic form as follows (Banker et.al [2]):
They demonstrated that DEA models can be used for efficiency evaluation of DMUs even the data set does not contain any explicit inputs. They formulated the index data model as follows:

\[
T = \{(X,Y) | X \geq \sum_{j=1}^{n} X_j \lambda_j, \ Y \leq \sum_{j=1}^{n} Y_j \lambda_j, \ \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \ j = 1, 2, \ldots, n\}.
\]  \tag{1}

The main aim of this paper is to formulate output-oriented DEA-R models and analyze their relationship to model (2).

Despic et al [4] used the DEA methodology and ratio analysis to develop DEA-R models for efficiency analysis of the DMUs under evaluation. They DEA-R model is formulated as follows:

\[
\begin{align*}
\text{Max} & \quad \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \frac{y_{rp}}{x_{ip}}, \\
\text{St} & \quad \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \frac{y_{rj}}{x_{ij}} \leq 1, \quad j = 1, 2, \ldots, n, \\
& \quad w_{ir} \geq 0, \quad i = 1, 2, \ldots, m, \ r = 1, 2, \ldots, s.
\end{align*}
\]  \tag{2}

Wei et al [14] applied the DEA-R model for evaluation of efficiency and pseudo-efficiency in health care (evaluation of hospitals). Further, Wei et al [15] discussed the efficiency underestimation of the CCR model. They used the DEA-R model in their research. In another work, Wei et al [16] compared the optimal weight of DEA and DEA-R models. Among other applications of DEA models in the last years (Bandyopadhyay[1]) can be mentioned.
Min \( \gamma_R \),

St

\[
\sum_{j=1}^{n} \lambda_j \left( \begin{array}{c} \frac{x_{ij}}{x_{ip}} \\ \frac{y_{ij}}{y_{ip}} \end{array} \right) \leq \gamma_R, \\
i = 1, 2, \ldots, m, \quad r = 1, 2, \ldots, s
\]  

(4)

\[
\sum_{j=1}^{n} \lambda_j = 1, \\
\lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\]

**Definition 2.** DMU\(_p\) is CCR-R-I efficient (input-oriented CCR-R efficient) if and only if the optimal objective function value of model (4) \( \gamma_R^* = 1 \).

### 2.2 AHP

AHP is one of the most efficient analytical hierarchy process decision-making techniques which were first introduced by Saaty [11]. This technique is based on paired comparison which allows managers to study various scenarios. In the science of decision, in which choosing one solution from the solutions at hand or prioritizing those solutions are considered, methods of MCDM (Multi Criteria Decision Making) especially AHP were rendered in recent years. Analytical hierarchy process (AHP) method is used more than any other method in Operations Research and has 4 principals:

1. Reciprocal Condition: If preference of element A upon element B is equal to \( n \), then the preference of element B upon element A will be equal to \( 1/n \).
2. Homogeneity: Element A and element B should be both equal and analogous. It means that the preference of element A upon B, cannot be zero or extreme.
3. Dependency: Each hierarchical element may depend on its upper element. This linear dependency can be continual until the highest level.
4. Expectations: Whenever a change or changes in hierarchical structures occur, the evaluation process should be repeated.

AHP has many properties; Unity and the uniqueness of the model, integration, balance and repetition are some of them.

Moreover, two important features which distinguish it from other techniques are:

1. Indicating accurate scales for fully compatible matrix of paired comparison.
2. Indicating the best local priorities, considering the judgment of decision-making units.

The first step in using AHP as a decision-making tool is to create an appropriate hierarchical tree which is in fact expressing the issue under study. In turning a matter of decision-making into a hierarchical matter, the overall objective is on the highest level; in the following level or levels there are criterions or sub-criterions. The final level includes the decision options. For example in choosing the best
contractor, first we separately compare candidature in terms of good experience, desirable quality, appropriate price, financial power, related equipment and skilled manpower. Then these features are compared to above mentioned criterions and their scales are indicated (figure1).

The next step is to form a paired comparison matrix. Here each level’s element is compared to its higher level element and their relative scales are indicated. We call these scales “relative scales”. Then with the combination of these relative scales, we can calculate each option’s final scale which is called the “absolute scale”.

Suppose ‘m’ is a decision criterion in $C_1, C_2, ..., C_m$ and $W$ is the vector from AHP method in $W = (W_1, W_2, ..., W_m)$which is true in these conditions: $\sum_j w_j = 0, j = 1, 2, ..., m$.

The paired comparison between “m” (decision criterion) is made by the decision maker or expert’s advice; the expert gives advice with regards to first the main goal of decision making, second the most important criterion and then which scales between 1 to 9 is appropriate for it.

Answering these questions organized a paired comparison matrix $m \times m$ like as $A$:

$$A = (a_{ij})_{m \times m} = \begin{bmatrix} C_1 & \vdots & \vdots & \vdots & \vdots \\ \vdots & a_{11} & ... & a_{1m} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & a_{m1} & \cdots & a_{mm} \end{bmatrix}$$

In which $a_{ij}$ indicates the judgment of $w_i/w_j$ with $a_{ij} = 1$ and $a_{ij} = 1/a_{ji}$ for each $i, j = 1, 2, ..., m$. Various methods have been presented in order to obtain the scale vector of $W$ from the paired comparison matrix $A$. The eigenvector method (developed by Prof. Saaty) is the most important among the all 4 types of scale calculation method in AHP, but there is no special consensus towards it. Here we present a brief description of eigenvector method:

Eigenvector method

In this method $w_i$ is indicated in a way that the following relations remain true:
\[ a_{11}w_1 + a_{12}w_2 + \cdots + a_{1n}w_n = \lambda w_1 \]
\[ a_{21}w_1 + a_{22}w_2 + \cdots + a_{2n}w_n = \lambda w_2 \]
\[ a_{n1}w_1 + a_{n2}w_2 + \cdots + a_{nn}w_n = \lambda w_n \]

And so on….

In these relations, \( a_{ij} \) is the preference of the in element upon \( jn \) and \( w_i \) is also the in’s scale and \( \lambda \) is a constant number.

This method is an average fraction kind. The above equations can be written down in a way that if \( A \) is a paired comparison matrix, then the scale vector of \( W \) can be obtain from solving \( AW = \lambda W \) equation. In this equation, \( \lambda \) is the largest amount of matrix \( A \). In eigenvector method, to calculate \( w_i \)'s, the following steps should be performed:

1. Create matrix \( A \).
2. Indicate \( A - \lambda I \) matrix.
3. Compute the determinant of the matrix \( A - \lambda I \) and make it equal to zero then compute the values of \( \lambda \).
4. Name the largest \( \lambda, \lambda_{\text{max}} \) and put it in \((A - \lambda_{\text{max}}I) \times W = 0\) equation, then calculate the \( w_i \) values.

The arithmetic mean method, which is one of the scale calculation methods, is applied in our calculations in this article. Steps in in this method are as follows:

First we sum each column’s values. Next, in order to normalize the paired comparison matrix, divide each element in paired comparison matrix on its column’s sum. Finally, compute the average value of the elements in the line from the normalized matrix. After computing the candidate’s scales toward the criterion, the final scale is obtained from the combination of candidate’s scale with criterion’s scale toward the target. Since the criterion’s scale reflect their importance in indicating the goal and each option’s scale toward criterion is its share from the related criterion, so the contractor’s final scale is obtained via multiplying each criterion’s scales in its related option scale from that criterion.

In regards to the above explanations, and by using Expert Choice software, after calculating the option’s scale and criterion’s scale, we indicate the final obtained prioritize.

3. Modified output-oriented DEA-R models

A mathematical model in DEA is created for judging the efficiency borders and it evaluates the decision making units if they are effective. Also DEA presents corrective solution for deficient units.

In order to set the new method, first calculate the column vector of arithmetic mean from paired comparison matrix and name it \( \overline{a}_i \). Then calculate the standard deviation of each line of normalized matrix (according to how Jahanshahloo et al [6]) and name it \( \delta_i \). In this new method, we consider each option or criterion in paired comparison matrix as DMU, so that \( \frac{1}{\delta_i} \) is regarded as the input and \( \overline{a}_i \) is regarded as the output. Like:

\[
DMU_1 = \left( \frac{1}{\delta_1}, \overline{a}_1 \right), \quad \ldots, \quad DMU_n = \left( \frac{1}{\delta_n}, \overline{a}_n \right)
\]

It means that there is one input and one output for each DMU. The multiple CCR model for evaluating the \( DMU_0 \) in the input place considering \( m \) as the input and \( s \) as the output is as follows:
Jahanshahloo et al [6] applied CCR as the input for computing the scales of paired comparison matrix. In this article considering an output and m as the input, a CCR turns into DEA-R model as follow; by changing the variables we have:

\[ u y_0 = \theta \rightarrow u = \frac{\theta}{y_0}, v x_0 = w \rightarrow v = \frac{w}{x_0} \]  

\[ \frac{\theta}{y_0} y_j - v x_j \leq 0, \theta \leq \frac{vx_j}{x_j/y_0} = \frac{wx_j}{x_0/y_j} = \frac{x_j/y_0}{x_0/y_0} \quad , \quad i = 1, \ldots, m \]

And CCR model [7] based on analysis deficit as the input is as follows:

\[ \text{max } \theta \]  

s.t. \[ \sum_{r=1}^{s} \sum_{j=1}^{m} w_{ij} \left( \frac{x_{ij}}{y_{ij}} \right) \geq \theta \quad (j = 1, \ldots, n) \]

\[ \sum_{i} \sum_{r} w_{ir} = 1 \quad w_{ir} \geq 0 \]

Therefore with the placement of inputs and outputs (1) in model (8), the following proposed model is obtained and the AHP’s scale is now calculable.

\[ \text{max } \theta \]  

s.t. \[ \sum_{r} \sum_{i} w_{ir} \left( \frac{1}{a_{ij}} \right) \geq \theta \quad (j = 1, \ldots, n) \rightarrow \]

\[ \sum_{r} \sum_{i} w_{ir} \left( \frac{a_{ij} \delta_{ij}}{a_{ij} \delta_{ij}} \right) \geq \theta \rightarrow \theta = \min \left| \frac{\delta_{ij}}{a_{ij} \delta_{ij}} \right| \]
Because $\sum_i \sum_r w_{ir} = 1$ and we have just one input and output, with turning the max function into min function and summarizing the above relations, the above correlation is proved which is in fact the DEA-R model.

4. Numerical illustration

We compared 6 candidatures who were accredited with regards to the criterion of good experience and financial power in Iranian Oil Pipeline and Telecommunication Company. Paired comparison matrix is as follows:

$$
A = \begin{bmatrix}
1 & 4 & 1 & 1 & 3 & 4 \\
1/4 & 1 & 7 & 3 & 1/5 & 1 \\
1/4 & 1/7 & 1 & 1/5 & 1/6 & 1 \\
1/3 & 5 & 1 & 1 & 1/3 & 1 \\
1/3 & 5 & 5 & 1 & 1 & 3 \\
1/4 & 1 & 6 & 3 & 1/3 & 1 \\
\end{bmatrix}
$$

Vector arithmetic mean and vector SD and normal matrix for the above matrix are as follow:

$$
\bar{a}_A = (0.283, \ 0.15, \ 0.064, \ 0.134, \ 0.221, \ 0.146)^T
$$

$$
\delta_A = (0.184, \ 0.122, \ 0.096, \ 0.093, \ 0.135, \ 0.11)^T
$$

$$
A = \begin{bmatrix}
.26 & .349 & .04 & .109 & .52 & .421 \\
.065 & .087 & .28 & .326 & .035 & .105 \\
.26 & .012 & .04 & .022 & .035 & .017 \\
.26 & .03 & .2 & .109 & .174 & .035 \\
.086 & .44 & .2 & .109 & .174 & .32 \\
.065 & .087 & .24 & .326 & .058 & .105 \\
\end{bmatrix}
$$

In the following tables, we compared results obtained from DEA-R method and Dr. Jahanshahloo et al method with the results obtained via the Expert Choice software.
Table 1

Comparison and ranking of DMU’s, considering good experience criterion with three different methods.

<table>
<thead>
<tr>
<th></th>
<th>Candidates’ weight.</th>
<th>Jahanshahloo model</th>
<th>E.C. Software</th>
<th>DEA-R model</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>w₁</td>
<td>1</td>
<td>0.289</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>DMU2</td>
<td>w₂</td>
<td>0.351</td>
<td>0.147</td>
<td>0.351</td>
<td>3</td>
</tr>
<tr>
<td>DMU3</td>
<td>w₃</td>
<td>0.118</td>
<td>0.057</td>
<td>0.118</td>
<td>6</td>
</tr>
<tr>
<td>DMU4</td>
<td>w₄</td>
<td>0.239</td>
<td>0.128</td>
<td>0.239</td>
<td>5</td>
</tr>
<tr>
<td>DMU5</td>
<td>w₅</td>
<td>0.573</td>
<td>0.236</td>
<td>0.573</td>
<td>2</td>
</tr>
<tr>
<td>DMU6</td>
<td>w₆</td>
<td>0.309</td>
<td>0.144</td>
<td>0.309</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2

Normal matrix of paired comparison of candidates, considering financial power with the new DEA-R method

<table>
<thead>
<tr>
<th></th>
<th>DMU1</th>
<th>DMU2</th>
<th>DMU3</th>
<th>DMU4</th>
<th>DMU5</th>
<th>DMU6</th>
<th>̅a</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>0.200</td>
<td>0.120</td>
<td>0.130</td>
<td>0.235</td>
<td>0.286</td>
<td>0.432</td>
<td>0.2340</td>
<td>0.0517</td>
</tr>
<tr>
<td>DMU2</td>
<td>0.200</td>
<td>0.120</td>
<td>0.130</td>
<td>0.176</td>
<td>0.143</td>
<td>0.108</td>
<td>0.1463</td>
<td>0.0158</td>
</tr>
<tr>
<td>DMU3</td>
<td>0.400</td>
<td>0.240</td>
<td>0.261</td>
<td>0.176</td>
<td>0.286</td>
<td>0.108</td>
<td>0.2452</td>
<td>0.0444</td>
</tr>
<tr>
<td>DMU4</td>
<td>0.050</td>
<td>0.040</td>
<td>0.087</td>
<td>0.059</td>
<td>0.071</td>
<td>0.027</td>
<td>0.0557</td>
<td>0.0097</td>
</tr>
<tr>
<td>DMU5</td>
<td>0.100</td>
<td>0.120</td>
<td>0.130</td>
<td>0.118</td>
<td>0.143</td>
<td>0.216</td>
<td>0.1379</td>
<td>0.0183</td>
</tr>
<tr>
<td>DMU6</td>
<td>0.050</td>
<td>0.120</td>
<td>0.261</td>
<td>0.235</td>
<td>0.071</td>
<td>0.108</td>
<td>0.1410</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

Table 3.

Comparison and ranking of DMU’s, considering financial power criterion with three different methods.

<table>
<thead>
<tr>
<th></th>
<th>Candidates’ weight.</th>
<th>Jahanshahloo model</th>
<th>E.C. Software</th>
<th>DEA-R model</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>w₁</td>
<td>1.00</td>
<td>0.249</td>
<td>1.00</td>
<td>1</td>
</tr>
<tr>
<td>DMU2</td>
<td>w₂</td>
<td>0.19</td>
<td>0.152</td>
<td>0.19</td>
<td>5</td>
</tr>
<tr>
<td>DMU3</td>
<td>w₃</td>
<td>0.90</td>
<td>0.257</td>
<td>0.90</td>
<td>2</td>
</tr>
<tr>
<td>DMU4</td>
<td>w₄</td>
<td>0.04</td>
<td>0.56</td>
<td>0.04</td>
<td>6</td>
</tr>
<tr>
<td>DMU5</td>
<td>w₅</td>
<td>0.21</td>
<td>0.146</td>
<td>0.21</td>
<td>4</td>
</tr>
<tr>
<td>DMU6</td>
<td>w₆</td>
<td>0.45</td>
<td>0.141</td>
<td>0.45</td>
<td>3</td>
</tr>
</tbody>
</table>
Then the ranking which is performed by Expert Choice software, which is specially designed to calculate and rank multi criteria decision making (MCDM) with AHP technique, is shown.

![Figure 2](image)

**Figure 2-** Comparison of candidates, considering financial power by *Expert Choice* software.

In the following figure, another method of calculating and final ranking of contractors is done by Expert Choice software.
5. Conclusions

This method is used as a mean to spread the ways of obtaining efficiency and having a better ranking based on one of the DEA’s model. Computing the efficiency is easy and there is no long computing in this method. According to the results, DMU’s ranking in this method is equal to Dr. Jahanshahloo’s method, since the numbers of inputs and outputs in both methods is equal and the way of computing is similar too. The general goal of applying these scientific and mathematical techniques is to increase the accuracy of decisions in management sciences. Moreover by these means, decision making process is made easier and complicated computing which may have the possibility of deficiency is removed. The presented DEA model not only computes the priorities of decision making units but also makes that task much easier and smoother than the other similar models. Although more investigation is needed to prove the applicability of this method, we have to use other utility computing software’s with this model especially when there are lots of inputs and outputs and this leads to a harder computing.

Acknowledgements
The research is supported by the Iranian Oil Pipelines and Telecommunication Company.

References