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Using MOLP based procedures to solve DEA problems

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Abstract

Data envelopment analysis (DEA) is a technique used to evaluate the relative efficiency of comparable decision making units (DMUs) with multiple input-output. It computes a scalar measure of efficiency and discriminates between efficient and inefficient DMUs. It can also provide reference units for inefficient DMUs without consideration of the decision makers' (DMs) preferences. In this paper, we deal with the problem of incorporating preferences over potential improvements to individual input output levels so that the resultant target levels reflect the DM's preferences over alternative paths to efficiency. In this way, the paper will establish an equivalence model between DEA and multiple objective linear programming (MOLP) and show how a DEA problem can be solved interactively by transforming it into an MOLP formulation. As a result, all efficient units of variable returns to scale technology in DEA can be found by solving the proposed MOLP problem by parametric linear programming. Numerical examples confirm the validity of the proposed model as a means for solving different DEA problems.

Keywords: Data envelopment analysis; Multiple objective linear programming; Additive model; Ecker-Kouada method; Most preferred solution

1. Introduction

Data envelopment analysis (DEA) is a technique used to evaluate the relative efficiency of comparable decision making units (DMUs) with multiple input-output. The DEA approach defines a non-parametric best practice frontier and then measures efficiency relative to that frontier. If a DMU lies on the frontier, it is referred to as an efficient unit, otherwise inefficient. It can also provide the reference units for inefficient DMUs without consideration of the decision makers' (DMs) preferences. For a managerial point of view, there may be a need sometimes to have a number of alternatives between which the decision maker has to decide. Reference units are composite or virtual units which lie on the efficient frontier and are used as target units for inefficient DMUs to benchmark against. Various techniques have been proposed for incorporating DM's preference information in DEA (e.g., see [3,4,12,…]). However, common to these approaches is that they all require a priori information from the DM, which in most cases can be subjective and difficult to obtain.

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An appealing method to incorporate preference information into both efficiency analysis and target setting, without necessarily requiring prior judgments, is to use of interactive multiple objective linear programming (MOLP) techniques. Golany [14] first proposed the use of an interactive procedure to generate efficient solutions for characterizing the efficient frontier in DEA.

Thanassoulis and Dyson [22] developed models for estimating preferred input-output target levels to make relatively inefficient organizational units efficient, allowing both input reductions and output increases. Stewart [21] contrasts the concept of relative efficiency in DEA with that of pareto optimality in MOLP and discusses some issues in applying interactive MOLP techniques to solve the weight restriction problem in DEA. Korhonen [18] provided an interactive method which allows the DM to incorporate preference information into the efficient frontier analysis by enabling him/her to make a free search various target units on the efficient frontier in DEA. Joro et al. [17] showed that structurally the DEA formulation to identify efficient units is quite similar to the MOLP model based on the reference point or the reference direction approach to generate efficient solutions. The reference point model enhances the usefulness of DEA by providing added flexibility to it. Post and Spronk [19] combined the use of DEA and interactive multiple goal programming where DMs preference information is incorporated interactively by adjusting the upper and lower feasible boundaries of the input and output levels. Yang et al. [26] investigated interactive MOLP methods to conduct efficiency analysis and set realistic target values in an integrated way with the DMs preferences taken into account and with the DM supported to explore what could be technically achievable. In this way, they established the equivalence relationship between the output-oriented DEA dual models and the minimax formulations led to the construction of the three equivalence models: namely the super-ideal point model, the ideal point model and the shortest distance model.

In a similar vein, the aim of this paper is to deal with the problem of incorporating preferences over potential improvements to individual input output levels so that the resultant target levels reflect the DM’s preferences over alternative paths to efficiency. In this way, the paper will establish the equivalence between the Additive model and the E-K formulation in MOLP and show how a DEA problem can be solved interactively without any prior judgment by transforming it into an MOLP formulation. To carry out our purpose, we use the method of Satisfactory Goals to reflect the DM's preferences in locating a most preferred solution (MPS) on the efficient frontier for target setting. As a result by solving the proposed MOLP problem, the efficient units of variable returns to scale technology in DEA can be obtained. The rest of this paper is organized as follows. Section 2 briefly reviews the basic DEA models used for this study. Section 3 provides the equivalence model between DEA and MOLP followed by some important results. Two illustrative examples are documented in section 4. A conclusion and future directions for research are all summarized in the last section.

2. Basic DEA models

Assume that there are n DMUs each consumes m inputs to produce s outputs. Let $X \in \mathbb{R}^{m\times n}$ and $Y \in \mathbb{R}^{s\times n}$ be the matrices consisting observed input and output measures for the DMUs, respectively. We denote by x_j (the jth column of X) the vector of inputs consumed by DMU_j. A similar notation is used for outputs. Also we assume that all the inputs and outputs are non-negative and at least one of the components of every input and output vector is positive.

In the original DEA model of Charnes, Cooper and Rhodes (CCR model [7]), the efficiency of a DMU can be obtained as the maximum of a ratio of weighted outputs to weighted inputs, subject to the condition that the same ratio for all DMUs must be less than or equal to one. The CCR model is based on constant returns to scale. However, in efficiency analysis, variable returns to scale (VRS) can also be considered. Banker et al. [5] proposed another version of DEA model with VRS, called the BCC model which we are interested in this paper.

The CCR and BCC models are the basic model types in DEA. These models can be presented in a primal or dual form, called multiplier and envelopment models, respectively. While the multiplier model can inform the DM of the efficiency score and the output and input weights, the envelopment model can be used to generate not only the efficiency score but also the lacks of outputs and the surplus of inputs of a unit. It can also provide reference units known as composite or virtual units which lie on the efficient frontier and are used as target units for inefficient DMUs to benchmark against.

Since, we are interested in BCC models in this paper, we represent the PPS of variable returns to scale (VRS) technology in the following manner:

$$T_v = \{(x, y) | x \geq X\lambda, y \leq Y\lambda, e\lambda = 1 , \lambda \geq 0\}$$

Where e is the sum vector of ones.

Based on the above definition, the envelopment form of BCC model is as follows:

$$\begin{aligned} \text{Min} \quad & \theta_o \\ s.t. \quad & X\lambda + S_o^- = \theta_o x_o \\ & Y\lambda - S_o^+ = y_o \\ & e\lambda = 1 \\ & \lambda \geq 0, S_o^- \geq 0, S_o^+ \geq 0 \end{aligned} \tag{1}$$

DMU$_o$ ($o \in \{1, \dots, n\}$) is strongly BCC-efficient if and only if $\theta_o^* = 1$ and the amounts of slacks be zero in any optimal solutions. In this paper, frontier DMUs that are not strongly efficient DMUs referred to as efficient one. More precisely "efficient" we use in this paper includes "weakly efficient".

The Additive model, proposed by Charnes et al. (20), is an alternative formulation to analyze the efficiency of DMUs which used the maximum distance between DMUs and the efficient frontier. There are several types of Additive models in DEA. The Additive model we select is

$$\begin{aligned} \text{Max} \quad & eS_o^- + eS_o^+ \\ s.t. \quad & X\lambda + S_o^- = x_o \\ & Y\lambda - S_o^+ = y_o \\ & e\lambda = 1 \\ & \lambda \geq 0, S_o^- \geq 0, S_o^+ \geq 0 \end{aligned} \tag{2}$$

A variant, which we do not explore here, is an Additive model which omits the condition $e\lambda = 1$.

As can be seen, the Additive model above has the same PPS as the BCC-model.

DMU_o is called ADD-efficient if and only if the optimal objective function value of model (2) is zero.

Theorem 1. DMU_o is ADD-efficient if and only if it is strongly BCC-efficient.

A proof of this theorem may be found in Ahn et al. [1].

Theorem 2. Let us define $\hat{x}_o = x_o - S_o^{*^-}$ and $\hat{y}_o = y_o + S_o^{*^+}$. Then $(\hat{x}_o, \hat{y}_o)$ is ADD-efficient.

Proof. See [11].

By this theorem, improvement to an efficient unit is attained by the following formulae (projection by the Additive model):

$$\begin{aligned} \hat{x}_o &= x_o - S_o^{*^-} = X\lambda^* \\ \hat{y}_o &= y_o + S_o^{*^+} = Y\lambda^* \end{aligned} \qquad (3)$$

With $(\hat{x}_o, \hat{y}_o)$ serving as the imaginary composite unit on the efficient frontier used to evaluate DMU_o. λ_j^* (jth element of λ^*) is the reference weight for DMU_j (j= 1,…,n) and $\lambda_j^* > 0$ means that DMU_j is used to construct the composite unit for DMU_o.

Note that such improvement strategy is imbedded in DEA a priori and does not necessarily take account of management preferences. In the following section, we will explore how DMs preferences can be incorporated into improvement strategies using interactive multiple objective optimization techniques.

3. MOLP based procedures for integrating efficiency analysis and target setting

In the Additive model (2), the reference set can be generated for inefficient DMU_s simultaneously by maximizing outputs and minimizing inputs in the sense of a vector optimization. So, such a problem can be regarded as a kind of multiple objective optimization problem. In this section, the equivalence relationship between the Additive DEA model and the E-K formulation in MOLP will be established. Also, we demonstrate that we get very useful results by combining both DEA and MOLP approaches.

An MOLP problem can be represented in a general form as follows:

$$\begin{aligned} \max \quad & h(\lambda) = [c^1\lambda, c^2\lambda, \ldots, c^m\lambda, c^{m+1}\lambda, \ldots, c^{m+s}\lambda] \\ s.t. \quad & \lambda\epsilon\Lambda = \{\lambda\epsilon\mathbb{R}^n \big| A\lambda = b, \lambda \geq 0, b \in \mathbb{R}^k\} \end{aligned} \qquad (4)$$

Where Λ is the feasible region in decision space and the constraint matrix $A \in \mathbb{R}^{k\times n}$ is of full row rank. In a more compact format, this MOLP is sometimes written

$$\max \quad \{C\lambda = z | \lambda \epsilon \Lambda\} \tag{5}$$

Where C is a (m+s)× n criterion matrix whose rows are the gradients c^i of the (m+s) objectives and z is the criterion vector (objective functions vector). Since multiple objective problems rarely have points that simultaneously maximize all of the objectives, the more generalized solution concept of efficiency is introduced.

Definition 1. $\bar{\lambda}\epsilon\Lambda$ is efficient iff there does not exist another $\lambda\epsilon\Lambda$ such that $C\lambda \geq C\bar{\lambda}, C\lambda \neq C\bar{\lambda}$. Otherwise, $\bar{\lambda}$ is inefficient.

The set of all efficient points is designated E and is called the efficient set. There are several methods for characterizing an efficient point in the MOLP problems. From Ecker and Kouada [13], we have the following theorem:

Theorem 3. Let λ_o be an extreme point of Λ and (6) be the E-K model.

$$\begin{aligned} \text{Max} \quad & e^t S \\ s.t. \quad & C\lambda - IS = C\lambda_o \\ & A\lambda = b \\ & 0 \leq \lambda \in \mathbb{R}^n \\ & 0 \leq S \in \mathbb{R}^{m+s} \end{aligned} \tag{6}$$

Then, (i) $\lambda_o \in E$ iff (6) has an optimal objective function value of zero, (ii) if (6) has a positive-unbounded objective function value, $E = \emptyset$, and (iii) if $(\bar{\lambda}, \bar{S})$ is an optimal solution of (6), $\bar{\lambda} \in E$.

Proof. (i) λ_o is an efficient extreme point of Λ iff there does not exist another $\lambda \in \Lambda$ such that $C\lambda \geq C\lambda_o$ and $C\lambda \neq C\lambda_o$. So by the first group of constraints there can not exist any feasible solution that makes $S \gneqq 0$. This fact implies that it is not possible for (6) to have a positive objective function value.

(ii) The dual of (6) is

$$\begin{aligned} \text{Min} \quad & (C\lambda_0)^t p + b^t y \\ s.t. \quad & C^t p + A^t y \geq 0 \\ & -Ip \geq e^t \\ & p, y \quad \text{unrestricted} \end{aligned} \tag{7}$$

Since the LP (6) is unbounded above, we conclude from the weak duality Theorem that the dual constraints in (7) must be inconsistent. But the dual constraints are independent of the vector λ_o and hence, if they are inconsistent for some λ_o, they remain inconsistent for any λ_o. We therefore conclude that (6) remains unbounded, even if we change λ_o, as long as it remains feasible. Thus, from (i), the proof is completed.

(iii) Suppose $\bar{\lambda} \notin E$. Then, there exists a $\hat{\lambda}\epsilon\Lambda$ such that $C\hat{\lambda} \geq C\bar{\lambda}$, $C\hat{\lambda} \neq C\bar{\lambda}$. With $C\hat{\lambda} - C\lambda_o = I\hat{S}$ and $C\bar{\lambda} - C\lambda_o = I\bar{S}$, we have $I\hat{S} \geq I\bar{S}$, $I\hat{S} \neq I\bar{S}$. Since $(\hat{\lambda}, \hat{S})$ contradicts the optimality of $(\bar{\lambda}, \bar{S})$, $\bar{\lambda}\epsilon E$.

Thus, the E-K model of (6) provides us with a means for finding an efficient point when $E \neq \emptyset$ and tells us that E is empty when no efficient points exist.

From the definition of feasible region Λ, the E-K model can be written as follows:

$$\begin{aligned} \text{Max} \quad & e^t S \\ s.t. \quad & C\lambda - IS = C\lambda_o \\ & \lambda \in \Lambda \\ & S \geq 0 \end{aligned} \tag{8}$$

From formulation (2), the Additive model can be equivalently rewritten as follows:

$$\begin{aligned} \text{M}ax \quad & eS_o^- + eS_o^+ \\ s.t. \quad & -X\lambda - S_o^- = -x_o \\ & Y\lambda - S_o^+ = y_o \\ & \lambda\epsilon\Lambda_o = \{\lambda | e\lambda = 1, \lambda \geq 0\} \\ & S_o^- \geq 0 \\ & S_o^+ \geq 0 \end{aligned} \tag{9}$$

Now, we show that formulation (9) is the same as formulation (8) under certain conditions. The purpose for establishing this equivalence is to use formulation (8) to conduct efficiency analysis, so that interactive MOLP techniques can be used to locate the MPS or set target values for the observed DMU_o.

Suppose the feasible decision space Λ in formulation (8) is set to be the same as defined in formulation (9), or $\Lambda = \Lambda_o$. Since λ_o is an extreme point of Λ_o, the set of column vectors of e corresponding to positive λ_o is a linearly independent set by definition. On the other hand, λ_o must be the oth unit vector in $\mathbb{R}^n$. Therefore, the Additive model given in (9) can be written as follows:

$$\begin{aligned} \text{M}ax \quad & eS_o^- + eS_o^+ \\ s.t. \quad & -X\lambda - S_o^- = -X\lambda_o \\ & Y\lambda - S_o^+ = Y\lambda_o \\ & \lambda\epsilon\Lambda_o = \{\lambda | e\lambda = 1, \lambda \geq 0\} \\ & S_o^- \geq 0 \\ & S_o^+ \geq 0 \end{aligned} \tag{10}$$

The equivalence relationship between the Additive model (2) or (9) and the E-K model (8) can be established by the following theorem.

Theorem 4. Suppose $C = \begin{bmatrix} -X_1 \dots -X_n \\ Y_1 \dots \ Y_n \end{bmatrix}$ and $S^t = [S^{-t}, S^{+t}]$ where $S^{-t} = (s_1^-, \dots, s_m^-)$ is the input excess vector and $S^{+t} = (s_1^+, \dots, s_s^+)$ is the output shortfall vector. The Additive model (9) can be equivalently transformed to the E-K model (8) using formulation (10) and the following equations:

$$\Lambda = \Lambda_o \quad (11)$$

$$S = S_o \quad (12)$$

Proof. Using (10)-(12), the Additive model can be equivalently rewritten as follows:

$$\begin{aligned} \text{Max} \quad & eS \\ s.t. \quad & C\lambda - IS = C\lambda_o \\ & \lambda \in \Lambda \\ & S \geq 0 \end{aligned}$$

The above model is the E-K model (8).

From theorem 4, we have the following results:

1. The above analyses show that the Additive model is actually constructed to locate a specific efficient solution, termed as DEA efficient solution on the efficient frontier of the following generic MOLP formulation

$$\begin{aligned} \text{Max} \quad & -X\lambda \\ \text{Max} \quad & Y\lambda \\ s.t. \quad & e\lambda = 1 \\ & \lambda \geq 0 \end{aligned} \quad (13)$$

that does not take account of management preferences into improvement strategies, so the efficient units in DEA can be obtained by solving formulation (13) (For more details see result 2). Hence an interactive tradeoff analysis procedure can be used for locating the MPS in DEA problems.

Since different DMUs have different preferences and relative weights for the objectives, and the weight λ in formulation (13) can not be set individually for each DMU, hence the following model can be constructed and solved to generate a locally most preferred solution for DMU_o:

$$\begin{aligned} \text{Max} \quad & -X\lambda \\ \text{Max} \quad & Y\lambda \\ s.t. \quad & X\lambda \leq x_o \\ & Y\lambda \geq y_o \\ & e\lambda = 1 \\ & \lambda \geq 0 \end{aligned} \quad (14)$$

The above model defines the production possibility set for the observed DMU_o, in which there may be more preferred efficient solutions than the DEA efficient solution.

In the following, we will describe an interactive procedure to use the above result to support the DM to search for the MPS. We emphasize that it is no restrict at all to use any interactive MOLP method.

An interactive MOLP procedure

Interactive procedures constitute techniques that allow the decision maker to explore along the efficient frontier so that he or she can reach the MPS. At each iteration, a solution, or group of solutions, is generated for examination. As a result of the examination, the decision maker inputs information to the solution procedure. In these methods assume that the DM is unable to indicate a priori preference information due to the complexity of the problems, but that he/she is able to give preference information on a local level to a particular solution. It enables the decision maker to learn more about his or her problem. Interactive procedures allow the DM to do what he or she does best (make improved judgment in the face of new information). Because of this, interactive methods are powerful tools for solving MOLP problems. Much work has been done on this class of methods. The interactive method of Satisfactory Goals is considered in this paper. To explain this method, let us consider the general MOLP formulation in (4).

The method of Satisfactory Goals, proposed by Benson [6], uses the "Bounded Objective Method" (see [16]) interactively to determine a satisfactory solution. In this method the DM specifies a set of acceptable initial goal levels $L_j, j = 1, \dots, m, m+1, \dots, m+s$ (which must be feasible) and then identifies one objective function whose goal level is the least satisfactory (LS). The LS objective is maximized subject to the original constraints and to additional constraints formed by the rest of the objective functions. The analyst, who works iteratively and interactively with the DM, can tighten one or more goal attainments until the optimal tradeoffs are achieved and thus the MPS is found maximizing the implicit utility function of the DM (taken from [16]). In the next section, a practical application is solved with this method.

2. A possible and currently popular way to generate efficient solutions of MOLP (13) is to use the weighted-sum approach. In this method, each objective is multiplied by a scalar weight. Then, the weighted objectives are summed to form a composite (or weighted-sums) objective function. Without loss of generality, we will assume that each weighting vector is normalized so that its elements sum to one. (taken from [20]). By solving the weighted-sums LP by parametric linear programming, all efficient units of variable returns to scale technology in DEA can be obtained. Since we consider the case where there is only one parameter α, the problem with one input and one output is discussed here.

This procedure finds all efficient DMUs only by solving one parametric linear programming problem as follows:

$$\begin{aligned} \text{Max} \quad & (1-\alpha)(-X\lambda) + \alpha(Y\lambda) \\ s.t. \quad & e\lambda = 1 \qquad (15) \\ & \lambda \geq 0 \end{aligned}$$

where $\alpha \in (-\infty, +\infty)$.

In this way, the purpose is to determine a set of efficient units T_v that is parametrically optimal as α goes from $-\infty \to +\infty$.

Although, solving a parametric linear programming problem needs more computational effort, but for large n (number of DMUs), solving a parametric problem may be more preferable than solving n linear programs. Therefore, the proposed model (15) can be used to generate all efficient units of T_v with one input and one output. Our claim will be further illustrated by a simple example in [11].

3. So far, we have shown that structurally the DEA formulation to identify efficient units is quite similar to a linear model (6) to generate efficient solutions in the MOLP problems. Now, we redefine the efficiency in DEA:

As discussed in section 2, DMU_o is ADD-efficient iff the optimal objective function value of model (2) is zero. This is equivalent to part (i) of Theorem (3). Hence, by Theorem (1), DMU_o is strongly BCC- efficient iff the E-K model has an optimal objective function value of zero. In other words, each pareto optimal solution of model (13) is corresponding to a strongly efficient production possibility in T_v and vice versa. Therefore, the efficient units in T_v can be characterized by solving an MOLP problem.

4. Numerical example

In this section, two numerical examples are examined to demonstrate the proposed interactive MOLP procedure to search for the MPS along the efficient frontier and illustrate the parametric linear programming problem to generate all efficient units of T_v.

4.1. Taking preferences into account in setting target values

A case study is carried out to demonstrate how performance assessment and target setting can be conducted in an integrated way using the interactive MOLP method investigated in the previous section. Consider 20 Iranian bank branches with three inputs and three outputs as shown in Table 1. The data set we used is from [2]. Note that the data are scaled. The Additive model is run to find the amounts of improvement needed for the inputs and outputs of each branch. As shown in Table (2), branches 2, 5, 6, 10, 11, 13, 14, 16 and 18 are found to be inefficient with respect to all 20 branches in the survey. For instance, the DEA composite unit of branch 2, which is an inefficient branch, can be represented as a linear combination of 0.5385 of branch 4, 0.4137 of branch 7 and 0.0487 of branch 15. In fact, the corresponding composite inputs are [0.796, 0.5898, 0.2794] and the corresponding composite outputs are [0.227, 0.7252, 0.8394]. However, the DM is not accepted the DEA composite input and output values as the MPS for branch 2. Hence, an interactive MOLP procedure is needed for searching the MPS along the efficient frontier. We use Satisfactory Goals method to reflect the DM's preferences in locating a MPS on the efficient frontier for target setting and resource allocation. An initial set of acceptable goal levels L^1= (-0.796, -0.5898, -0.2794, 0.227, 0.7252, 0.8394) is given by the DM. The DM does not agree with the value of -0.796 for f_1 and wants the first input be at most 0.7. With the help of the dual variables associated with the f_3, f_5 and f_6 ([0, -0.4772, 0, -0.1773, -0.4276]), he / she revises some goals and sets new goals, L^2= (-0.796, -0.5898, -0.3465, 0.227, 0.5447, 0.7646). The optimized f_1 is -0.7285 and the composite input and output values for branch 2 are as (0.7285, 0.5898, 0.3465, 0.227, 0.627, 0.7973). DM is still not satisfied with the target level for the first input at 0.7285. With the help of the dual variables, he / she agrees to the degradation of f_2 and f_3 simultaneously so as to increase the current value of f_1 to -0.6. The approaching goal of 0.1285=(-0.6+0.7285) is divided in two ($\frac{0.1285}{2}$=0.0643) and distributed for f_2, Δf_2=-0.0211(=$\frac{0.0643}{-3.0495}$) and for f_3, , Δf_3=-0.1298(=$\frac{0.0643}{-0.4953}$). The new set of goals is L^3= (-0.7285, -0.5898-0.0211, -0.3465-0.1298, 0.227, 0.5447, 0.7646)= (-0.7285, -0.6109, -0.4763, 0.227, 0.5447, 0.7646).

Using this new set, the optimized f_1 is calculated as -0.6941. In iteration 3, the composite unit of branch 2 is given by (0.6941, 0.5941, 0.3894, 0.227, 0.627, 0.7646). Note that the new "Learnt" target level is different from the previous target level in the second interaction. The change in the target level made by the DM represents part of the learning process about what could be achieved, which is the main features of the interactive procedure and can help the DM to set realistic target values. The DM indicates the attained value of f_1 is satisfactory and that it can be relaxed to improve the second output. He specifies Δf_1=0.0059 (i.e., f_1 can be relaxed up to -0.7). Using the new goal level for f_1 at -0.7, we obtain the maximum composite output 2 as 0.6421. The DM is satisfied with this value and does not want to relax it. This means the MPS has been found and hence we terminate the procedure. The final composite unit can he presented as "0.2921 branch 4 + 0.1879 branch 7 + 0.089 branch 15 + 0.431 branch 20" with the input and output values as follows:

$$(I_1, I_2, I_3, O_1, O_2, O_3) = (0.7, 0.595, 0.3827, 0.227, 0.6421, 0.7646)$$

Now, the interactive procedure is demonstrated for the fifth DMU, which is operating as an inefficient branch too. The DEA target unit for inefficient branch 5 is as a linear combination of 0.5176 of branch 4, 0.2429 of branch 7, 0.1718 of branch 12, 0.0678 of branch 15 with the following input and output values:

$$(I_1, I_2, I_3, O_1, O_2, O_3) = (0.8081, 0.6064, 0.268, 0.233, 0.722, 0.806)$$

The DM is not satisfied with this initial target values and he prefers to improve the second output to above a value of 0.76 at the expense of f_2 and f_6. In fact, both the output level of charge and the input level of computer terminal are sacrificed for a higher output level of loan. In the first iteration, f_6 is decreased by about Δf_6=0.506 resulting in L^2= (-0.8081, -0.6064, -0.268, 0.233, 0.722, 0.3). for the second iteration, the DM is satisfied with the value of -0.603 for the second input and wants to relax it to -0.7 (Δf_2=0.097) to improve the second output. In this step, the DM selects f_5 (second output) from L^3= (-0.8081, -0.7, -0.268, 0.233, 0.722, 0.3) as f_{LS}. The maximum feasible value of the second composite output of branch 5 is generated as 0.8216 resulting in the new target (0.8081, 0.6801, 0.268, 0.233, 0.8216, 0.4688). DM agrees with the relaxation of Δf_5=0.0216 so as to decrease the first input. So, the new goal level L^4= (-0.8081, -0.7, -0.268, 0.233, 0.8, 0.3) is used for the next iteration. The Optimized f_1 with this new set of goals is -0.7838 and the dual values of f_3 and f_5 against f_1 are -2.9427 and -1.1264 respectively. The DM does not agree with the value of -0.7838 for f_1 and wants f_1 be at least -0.75. With the help of the dual variables, he revises some goals and sets new goals, L^5= (-0.7838, -0.7, -0.2822, 0.233, 0.7628, 0.3), leading to the new target (0.7469, 0.6803, 0.268, 0.233, 0.7628, 0.5121). However, DM accepted this solution as the MPS for branch 5 and hence the interactive method terminated. The MPS could be found for all inefficient branches using the interactive procedure. It can be observed that these inefficient branches have different MPS and DEA composite inputs and outputs.

4.2. Finding efficient units in DEA

Consider eight DMUs with one input and one output as listed in Table 3. The data reported here are from [11]. By employing the proposed model (14), we have the DMUs A, B, C, E and H are efficient as portrayed in Figure 1. Note model (14) also provides the range of values of α within which each DMU remains efficient as shown in the forth column of Table 3. As can be seen, all efficient DMUs of T_v are identified by solving one parametric linear programming problem.

Table 1: Data set for 20 branches of bank

Branch	Inputs			Outputs		
	Staff	Computer terminals	Space (m2)	Deposits	Loans	Charge
DMU1	0.950	0.700	0.155	0.190	0.521	0.293
DMU2	0.796	0.600	1.000	0.227	0.627	0.462
DMU3	0.798	0.750	0.513	0.228	0.970	0.261
DMU4	0.865	0.550	0.210	0.193	0.632	1.000
DMU5	0.815	0.850	0.268	0.233	0.722	0.246
DMU6	0.842	0.650	0.500	0.207	0.603	0.569
DMU7	0.719	0.600	0.350	0.182	0.900	0.716
DMU8	0.785	0.750	0.120	0.125	0.234	0.298
DMU9	0.476	0.600	0.135	0.080	0.364	0.244
DMU10	0.678	0.550	0.510	0.082	0.184	0.049
DMU11	0.711	1.000	0.305	0.212	0.318	0.403
DMU12	0.811	0.650	0.255	0.123	0.923	0.628
DMU13	0.659	0.850	0.340	0.176	0.645	0.261
DMU14	0.976	0.800	0.540	0.144	0.514	0.243
DMU15	0.685	0.950	0.450	1.000	0.262	0.098
DMU16	0.613	0.900	0.525	0.115	0.402	0.464
DMU17	1.000	0.600	0.205	0.090	1.000	0.161
DMU18	0.634	0.650	0.235	0.059	0.349	0.068
DMU19	0.372	0.700	0.238	0.039	0.190	0.111
DMU20	0.583	0.550	0.500	0.110	0.615	0.764

Table 2:

DEA composite unite results

	1	3	4	7	8	9	12	15	17	19	20
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0	0.5385	0.4137	0	0	0	0.0478	0	0	0
3	0	1	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0
5	0	0	0.5176	0.2429	0	0	0.1718	0.0678	0	0	0
6	0	0	0.8469	0.1339	0	0	0	0.0192	0	0	0
7	0	0	0	1	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0
10	0	0	0.3369	0	0	0	0	0	0	0	0.6631
11	0	0	0.1651	0.6598	0	0.1251	0	0.0501	0	0	0
12	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	0.6663	0	0.161	0	0.0257	0	0	0.147
14	0	0	1	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	1	0	0	0
16	0	0	0	0.2206	0	0	0	0	0	0	0.7794
17	0	0	0	0	0	0	0	0	1	0	0
18	0	0	0.1478	0.4135	0	0.4386	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	1	0
20	0	0	0	0	0	0	0	0	0	0	1

Table 3:

DMUs data and results of model (14) for Example 4.2

DMU	Input	Output	Characteristic interval
A	2	1	$[1, +\infty)$
B	3	3	$[0.4, 0.5]$
C	2	2	$[0.5, 1]$
D	4	3	
E	6	5	$[0, 0.4]$
F	5	2	
G	6	3	
H	8	5	$(-\infty, 0]$

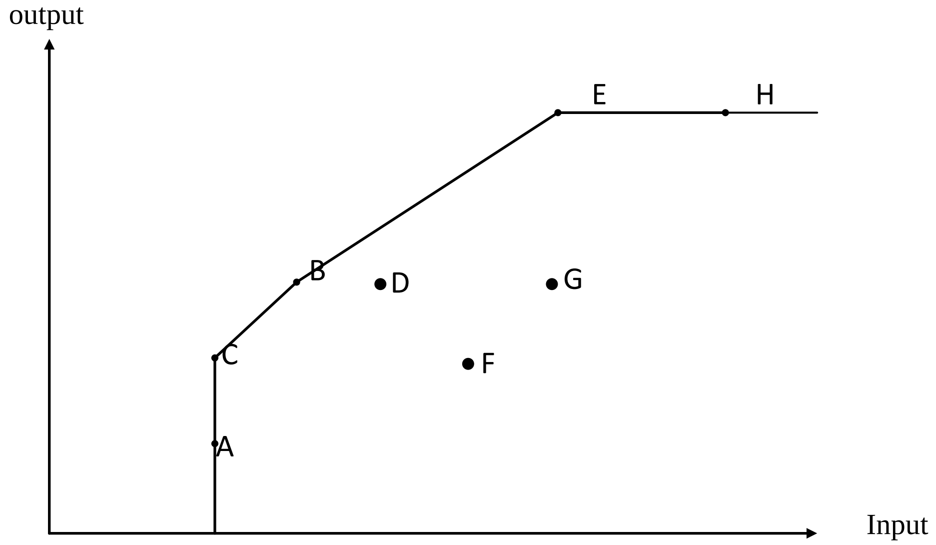

Figure 1. Efficient frontier of T_v

5. Conclusion

In this paper, we established an equivalence model between DEA and MOLP and show how a DEA problem can be solved interactively by transforming it into an MOLP formulation. This provides a basis to apply interactive methods and other techniques in MOLP to solve DEA problems. In this way, the use of the interactive Satisfactory Goals method for target setting was considered. An example on the assessment of Iranian bank branches was shown to illustrate the equivalence model and interactive procedure to search for MPS along the efficient frontier. In addition, an analysis of results led to finding efficient units on the frontier of T_v. Using other algorithms in characterizing the efficient set of an MOLP problem for conducting an interactive tradeoff analysis procedure in DEA is an interesting and useful issue for future research.

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